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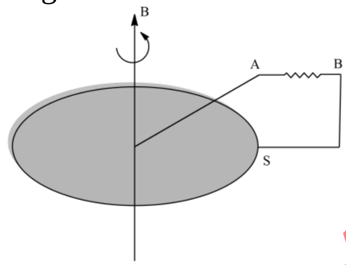
CSIR-UGC-NET/JRF DEC-2013

PHYSICAL SCIENCES DEC 2013

PART–B

21. A horizontal metal disc rotates about the vertical axis in a uniform magnetic field point up as shown in the figure. A circuit is made by connecting one end A of a resistor to the centre of the disc and the other end B to its edge through a sliding contract. S the current that shows through the resistor is



(a) Zero (b) DC from A to B
(c) DC from B to A (d) AC

22. A spin $-\frac{1}{2}$ particle is in the state $\chi = \frac{1}{\sqrt{11}}\begin{pmatrix} 1+i \\ 3 \end{pmatrix}$ in the eigenbasis of S^2 and S_z. If we measure S_z the probabilities of getting $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$ respectively are

(a) $\frac{1}{2}$ and $\frac{1}{2}$ (b) $\frac{2}{11}$ and $\frac{9}{11}$ (c) 0 and 1 (d) $\frac{1}{11}$ and $\frac{3}{11}$

23. Which of the following functions cannot be the real part of a complex analytic function of $z = x + iy$?

(a) $x^2 y$ (b) $x^2 - y^2$ (c) $x^3 - 3xy^2$ (d) $3x^2 y - y - y^3$

24. The motion of a particle of mass m in one dimension is described by the Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \lambda x$. What is the difference between the (quantized) energies of the first two levels? (In the following, $\langle x \rangle$ is the expectation value of x in the ground state)

(a) $\hbar\omega - \lambda(x)$ (b) $\hbar\omega + \lambda(x)$ (c) $\hbar\omega + \frac{\lambda^2}{2m\omega^2}$ (d) $\hbar\omega$

25. Let Ψ_{nlm} denote the eigenfunction of a Hamiltonian for a spherically symmetric potential V(r). The expectation value of L_z in the state.

$\Psi = \frac{1}{6}\left[\Psi_{200} + \sqrt{5}\Psi_{210} + \sqrt{10}\Psi_{21-1} + \sqrt{20}\Psi_{211}\right]$ is

(a) $-\frac{5}{18}\hbar$ (b) $\frac{5}{6}\hbar$ (c) $\hbar$ (d) $\frac{5}{18}\hbar$

26. Three identical spin $-\frac{1}{2}$ fermions are to be distributed in two non-degenerate distinct energy levels. The number of ways this can be done is

(a) 8 (b) 4 (c) 3 (d) 2

27. Let A, B and C be functions of phase space variables (coordinates and momenta of a mechanical system). If {,} represents the Poisson bracket, the value of $\{A,\{B,C\}\}-\{\{A,B\},C\}$ is given by

(a) 0 (b) $\{B,\{C,A\}\}$ (c) $\{A,\{C,B\}\}$ (d) $\{\{C,A\},B\}$

28. If A, B and C are non-zero Hermitian operators, which of the following relations must be false?

(a) $[A,B]=C$ (b) $AB+BA=C$ (c) $ABA=C$ (d) $A+B=C$

29. The expression

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}+\frac{\partial^2}{\partial x_4^2}\right)\frac{1}{\left(x_1^2+x_2^2+x_3^2+x_4^2\right)}$$ is proportional to

(a) $\delta(x_1+x_2+x_3+x_4)$ (b) $\delta(x_1)\delta(x_2)\delta(x_3)\delta(x_4)$

(c) $\left(x_1^2+x_2^2+x_3^2+x_4^2\right)^{-3/2}$ (d) $\left(x_1^2+x_2^2+x_3^2+x_4^2\right)^{-2}$

30. Given that the integral $\int_0^\infty \frac{dx}{y^2+x^2}=\frac{\pi}{2y}$, the value of $\int_0^\infty \frac{dx}{\left(y^2+x^2\right)^2}$ is

(a) $\frac{\pi}{y^3}$ (b) $\frac{\pi}{4y^3}$ (c) $\frac{\pi}{8y^3}$ (d) $\frac{\pi}{2y^3}$

31. The force between two long and parallel wires carrying currents I_1 and I_2 and separated by a distance D is proportional to

(a) I_1I_2/D (b) $(I_1+I_2)/D$ (c) $(I_1I_2/D)^2$ (d) I_1I_2/D^2

32. A loaded dice has the probabilities $\frac{1}{21},\frac{2}{21},\frac{3}{21},\frac{4}{21},\frac{5}{21}$ and $\frac{6}{21}$ of turning up 1, 2, 3, 4, 5 and 6, respectively. If it is thrown twice, what is the probability that the sum of the numbers that turn up is even?

(a) $\frac{144}{441}$ (b) $\frac{225}{441}$ (c) $\frac{221}{441}$ (d) $\frac{220}{441}$

33. A particle moves in a potential $V=x^2+y^2+\frac{z^2}{2}$. Which component (s) of the angular momentum is /are constants of motion?

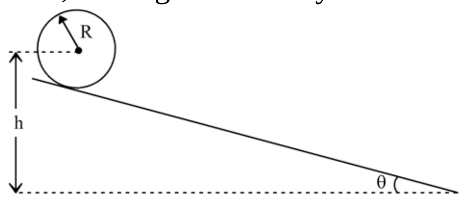
(a) None (b) L_x, L_y and L_z (c) only L_x and L_y (d) only L_z

34. The Hamiltonian of a relativistic particle of rest mass m and momentum p is given by $H=\sqrt{p^2+m^2}+V(x)$, in units in which the speed of light $c=1$. The corresponding Lagrangian is

(a) $L=m\sqrt{1+\dot{x}^2}-V(x)$ (b) $L=-m\sqrt{1-\dot{x}^2}-V(x)$

(c) $L=\sqrt{1+m\dot{x}^2}-V(x)$ (d) $L=\frac{1}{2}m\dot{x}^2-V(x)$

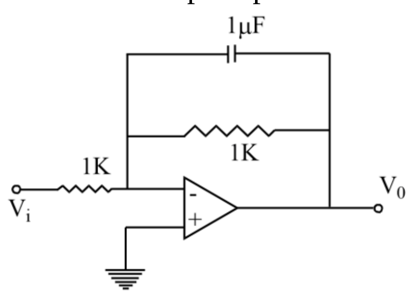
35. A ring of mass m and radius R rolls (without slipping) down an inclined plane starting from rest. If the centre of the ring is initially at a height h, the angular velocity when the ring reaches the base is



(a) $\sqrt{g/(h-R)}\tan\theta$ (b) $\sqrt{g/(h-R)}$

(c) $\sqrt{g(h-R)/R^2}$ (d) $\sqrt{2g/(h-R)}$

36. Consider the op-amp circuit shown in the figure



If the input is a sinusoidal wave $V_i = 5\sin(1000t)$, then the amplitude of the output V_0 is

(a) $\frac{5}{2}$ (b) 5 (c) $\frac{5\sqrt{2}}{2}$ (d) $5\sqrt{2}$

37. If one of the inputs of a J-K flip flop is high and the other is low, then the outputs Q and $\bar{Q}$

(a) Oscillate between low and high in race-around condition
(b) Toggle and the circuit acts like a T flip flop
(c) Are opposite to the inputs
(d) Follow the inputs and the circuit acts like an R-S flip flop

38. Two monochromatic sources, L_1 and L_2, emit at 600 and 700 nm, respectively. If their frequency bandwidths are 10^{-1} and 10^{-3} GHz, respectively, then the ratio of line width of L_1 and L_2 is approximately

(a) 100 : 1 (b) 1 : 85 (c) 75 : 1 (d) 1 : 75

39. Let (V, A) and (V', A') denote two sets of scaler and vector potentials, and Ψ a scalar function which of the following transformations leave the electric and magnetic fields (and hence Maxwell's equations unchanged?

(a) $A' = A + \nabla\Psi$ and $V = V - \frac{\partial\Psi}{\partial t}$ (b) $A' = A - \nabla\Psi$ and $V' = V + 2\frac{\partial\Psi}{\partial t}$

(c) $A' = A + \nabla\Psi$ and $V' = V + \frac{\partial\Psi}{\partial t}$ (d) $A' = A - 2\nabla\Psi$ and $V' = V - \frac{\partial\Psi}{\partial t}$

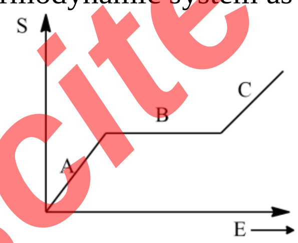
40. Consider the melting transition of ice into water at constant pressure. Which of the following thermodynamic quantities does not exhibit a discontinuous change across the phase transition?
(a) Internal energy (b) Helmholtz free energy
(c) Gibbs free energy (d) entropy

PART-C

41. Two different thermodynamic systems are described by the following equations of state:
$\frac{1}{T^{(1)}}=\frac{3RN^{(1)}}{2U^{(1)}}$ and $\frac{1}{T^{(2)}}=\frac{5RN^{(2)}}{2U^{(2)}}$ where $T(1,2)$, $N^{(1,2)}$ and $U^{(1,2)}$ are respectively, the temperatures, the mole numbers and the internal energies of the two systems, are R is the gas constant. Let U_{total} denote the total energy when these two systems are put in contact and attain thermal equilibrium. The ratio $\frac{U^{(1)}}{U_{tot}}$ is
(a) $\frac{5N^{(2)}}{3N^{(1)}+5N^{(2)}}$ (b) $\frac{3N^{(1)}}{2n^{(1)}+5N^{(2)}}$ (c) $\frac{N^{(1)}}{N^{(1)}+N^{(2)}}$ (d) $\frac{N^{(2)}}{N^{(1)}+N^{(2)}}$

42. The speed v of the molecules of mass m of an ideal gas obeys Maxwell's velocity distribution law at an equilibrium temperature T. Let (v_x,v_y,v_z) denote the components of the velocity and k_B the Boltzmann constant. The average value of $(\alpha v_x-\beta v_y)^2$, where α and β are constants is
(a) $(\alpha^2-\beta^2)k_\beta T/m$ (b) $(\alpha^2+\beta^2)k_\beta T/m$
(c) $(\alpha+\beta)^2 k_\beta T/m$ (d) $(\alpha-\beta)^2 k_\beta T/m$

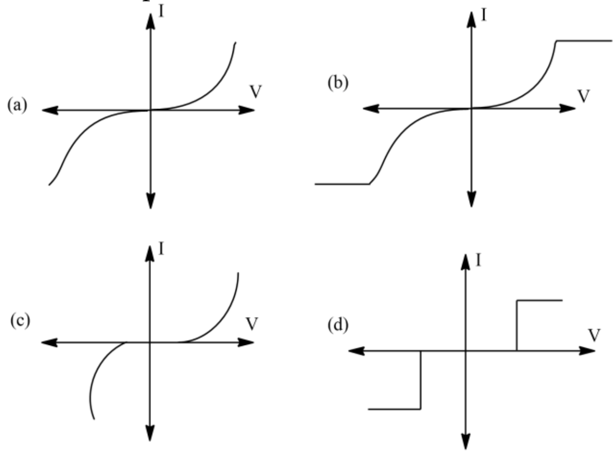
43. The entropy S of a thermodynamic system as a function of energy E is given by the following graph



The temperatures of the phases A, B and C denoted by T_A, T_B and T_C, respectively, satisfy the following inequalities:
(a) $T_C>T_B>T_A$ (b) $T_A>T_C>T_B$ (c) $T_B>T_C>T_A$ (d) $T_B>T_A>T_C$

44. The Physical phenomenon that cannot be used for memory storage applications is
(a) Large variation in magnetoresistance as a function of applied magnetic field
(b) Variation in magnetization of a ferromagnet as a function of applied magnetic field
(c) Variation in polarization of a ferroelectric as a function of applied electric field
(d) Variation in resistance of a metal as a function of applied electric field

45. Two identical Zener diodes are placed back in series and are connected to a variable DC power supply. The best representation of the I-V characteristics of the circuit is



46.A pendulum consists of a ring of mass M and radius R suspended by a massless rigid rod of length l attached to its rim. When the pendulum oscillates in the plane of the ring, the time period of oscillation is

(a) $2\pi\sqrt{\frac{l+R}{g}}$ (b) $\frac{2\pi}{\sqrt{g}}\left(l^2+R^2\right)^{1/4}$ (c) $2\pi\sqrt{\frac{2R^2+2Rl+l^2}{g(R+l)}}$ (d) $\frac{2\pi}{\sqrt{g}}\left(2R^2+2Rl+l^2\right)^{1/4}$

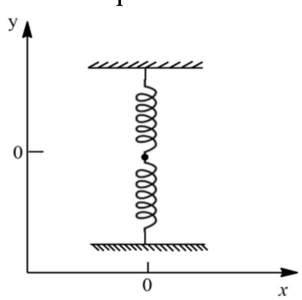
47.Spherical particles of a given material of density p are released from rest inside a liquid medium of lower density. The viscous drag force may be approximated by the Stoke's law, i.e., $F_d = 6\pi\eta Rv,$ where η is the viscosity of the medium, R the radius of a particle and v its instantaneous velocity. If $\tau(m)$ is the time taken by a particle of mass m to each half its terminal velocity, then the ratio $\tau(8m)/\tau(m)$ is

(a) 8 (b) 1/8 (c) 4 (d) ¼

48.A system of N classical non-interacting particles, each of mass $m,$ is at a temperature T and is confined by the external potential $V(r)=\frac{1}{2}Ar^2$ (where A is a constant) in three dimensions. The internal energy of the system is

(a) $3Nk_BT$ (b) $\frac{3}{2}Nk_BT$ (c) $N(2mA)^{3/2}k_BT$ (d) $N\sqrt{\frac{A}{m}}\ln\left(\frac{k_BT}{m}\right)$

49.Consider a particle of mass m attached to two identical springs each of length l and spring constant k (see the figure below). The equilibrium configuration is the one where the3 springs are unstretched. There are no other external forces on the system. If the particle is given a small displacement along the x-axis, which of the following described the equation of motion for small oscillations?



(a) $m\ddot{x}+\frac{kx^3}{l^2}=0$ (b) $m\ddot{x}+kx=0$ (c) $m\ddot{x}+2kx=0$ (d) $m\ddot{x}+\frac{kx^2}{l}=0$

50.If $\Psi(x)=A\exp(-x^4)$ is the eigenfunction of a one dimensional Hamiltonian with eigenvalue E =0, the potential $V(x)$ (in units where $\hbar=2m=1$) is

(a) $12x^2$ (b) $16x^6$ (c) $16x^6+12x^2$ (d) $16x^6-12x^2$

51.The electric field of an electromagnetic wave is given by $\bar{E}=E_0\cos\left[\pi(0.3x+0.4y-1000t)\right]\hat{k}$. The associated magnetic field $\bar{B}$ is

(a) $10^{-3}E_0\cos\left[\pi(0.3x+0.4y-1000t)\right]\hat{k}$ (b) $10^{-4}E_0\cos\left[\pi(0.3x+0.4y-1000t)\right](4\hat{i}-3\hat{j})$

(c) $E_0\cos\left[\pi(0.3x+0.4y-1000t)\right](0.3\hat{i}+0.4\hat{j})$ (d) $10^2E_0\cos\left[\pi(0.3x+0.4y-1000t)\right](3\hat{i}+4\hat{j})$

52. The energy of an electron in a band as a function of its wave vector k is given by $E(k)=E_0-B(\cos k_xa+\cos k_ya+\cos k_za)$, where E_0, B and 'a' are constants. The effective mass of the electron near the bottom of the band is

(a) $\frac{2\hbar^2}{3Ba^2}$ (b) $\frac{\hbar^2}{3Ba^2}$ (c) $\frac{\hbar^2}{2Ba^2}$ (d) $\frac{\hbar^2}{Ba^2}$

53.A DC voltage V is applied across a Josephson junction between two superconductors with a phase difference ϕ_0. If I_0 and k are constants that depend on the properties of the junction, the current flowing through it has the form

(a) $I_0\sin\left(\frac{2eVt}{\hbar}+\phi_0\right)$ (b) $kV\sin\left(\frac{2eVt}{\hbar}+\phi_0\right)$ (c) $kV\sin\phi_0$ (d) $I_0\sin\phi_0+kV$

54. Consider the following ratios of the partial decay widths $R_1=\frac{\Gamma(\rho^+\to\pi^++\pi^0)}{\Gamma(\rho^-\to\pi^-+\pi^0)}$ and $R_2=\frac{\Gamma(\Delta^{++}\to\pi^++p)}{\Gamma(\Delta^-\to\pi^-+n)}$. If the effect of electromagnetic and weak interactions are neglected, then R_1 and R_2 are respectively,

(a) 1 and $\sqrt{2}$ (b) 1 and 2 (c) 2 and 1 (d) 1 and 1

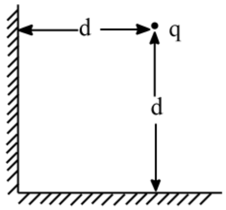
55. The intrinsic electric dipole moment of a nucleus ${}^A_ZX$

(a) Increases with Z, but independent of A
(b) Decreases with Z, but independent of A
(c) Is always zero
(d) Increases with Z and A

56. According to the shell model, the total angular momentum (in units of $\hbar$) and the parity of the ground state of the ${}^{7}_{3}Li$ nucleus is

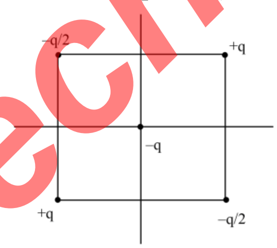
(a) $\frac{3}{2}$ with negative parity
(b) $\frac{3}{2}$ with positive parity
(b) $\frac{1}{2}$ with positive parity
(d) $\frac{7}{2}$ with negative parity

57. A point charge q is placed symmetrically at a distance infinite plates as shown in the figure. The net force on the charge (in units $1/4\pi\in_0$) is



(a) $\frac{q^2}{8d^2}\left(2\sqrt{2}-1\right)$ away from the corner
(b) $\frac{q^2}{8d^2}\left(2\sqrt{2}-1\right)$ towards the corner
(c) $\frac{q^2}{2\sqrt{2}d^2}$ towards the corner
(d) $\frac{3q^2}{8d^2}$ away from the corner

58. Let four point charges $q, -q/2, q$ and $-q/2$ be placed at the vertices of a square of side a. Let another point charge $-q$ be placed at the centre of the square (see the figure).



Let $V(r)$ be the electrostatic potential at a point P at a distance r >> a from the centre of the square. Then $V(2r)/V(r)$ is

(a) 1
(b) 1/2
(c) 1/4
(d) 1/8

59. Let A and B be two vectors in three-dimensional Euclidean space. Under rotation, the tensor product $T_{ij} = A_i B_j$

(a) Reduces to a direct sum of three 3-dimensinla representations
(b) Is an irreducible 9-dimensionla representation
(c) Reduces to a direct sum of a 1-dimensinal, a 3-dimensinla and a 5-dimensional irreducible representation
(d) Reduces to a direct sum of a 1-dimensional and an 8-dimensional irreducible representation

60. Fourier transform of the derivative of the Dirac $\delta-$function, namely $\delta'(x)$, is proportional to

(a) 0
(b) 1
(c) sink
(d) ik

61. A particle is in the ground state of an infinite square well potential given by,

$$V(x)=\begin{cases}0 & for\ -a\le x\le a\\ \infty & otherwise\end{cases}$$

The probability to find the particle in the interval between $-\frac{a}{2}$ and $\frac{a}{2}$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{2}+\frac{1}{\pi}$ (c) $\frac{1}{2}-\frac{1}{\pi}$ (d) $\frac{1}{\pi}$

62. The expectation value of the x-component of the orbital angular momentum L_x in the state $\Psi=\frac{1}{5}\left[3\Psi_{2,1,-1}+\sqrt{5}\Psi_{2,1,0}-\sqrt{11}\Psi_{2,1,+1}\right]$ (where Ψ_{nlm} are the eigenfunctions in usual notation), is

(a) $-\frac{\hbar\sqrt{10}}{25}\left(\sqrt{11}-3\right)$ (b) 0

(c) $\frac{\hbar\sqrt{10}}{25}\left(\sqrt{11}+3\right)$ (d) $\hbar\sqrt{2}$

63. A particle is prepared in a simultaneous eigenstates of L^2 and L_z. If $\ell(\ell+1)\hbar^2$ and $m\hbar$ are respectively the eigenvalue of L^2 and L_z then the expectation value $\langle L_x^2\rangle$ of the particle in this state satisfies

(a) $\langle L_x^2\rangle=0$ (b) $0\le\langle L_x^2\rangle\le\ell^2\hbar^2$

(c) $0\le\langle L_x^2\rangle\le\frac{\ell(\ell+1)\hbar^2}{3}$ (d) $\frac{\ell\hbar^2}{2}\le\langle L_x^2\rangle\le\frac{\ell(\ell+1)\hbar^2}{2}$

64. If the electrostatic potential $V(r,\theta,\phi)$ in a charge free region has the form $V(r,\theta,\phi)=f(r)\cos\theta$, then the functional form of $f(r)$ (in the following a and b are constants)

(a) $ar^2+\frac{b}{r}$ a (b) $ar+\frac{b}{r^2}$ (c) $ar+\frac{b}{r}$ (d) $a\ln\left(\frac{r}{b}\right)$

65. If $A=\hat{i}yz+\hat{j}xz+kxy$, then the integral $\oint_C A.dl$ (where C is along the perimeter of a rectangular) area bounded by $x=0,\ \ x=a$ and $y=0,\ \ y=b$) is

(a) $\frac{1}{2}\left(a^3+b^2\right)$ (b) $\pi\left(ab^2+a^2b\right)$ (c) $\pi\left(a^3+b^3\right)$ (d) 0

66. Consider an $n\times n(n>1)$ matrix A, in which Aij is the product of the indices i and j (namely A_{ij} = ij). The matrix A

(a) Has one degenerate eigenvalue with degeneracy $(n-1)$

(b) Has two degenerate eigenvalues with degeneracy's 2 and $(n-2)$

(c) Has one degenerate eigenvalue with degeneracy n

(d) Does not have any degenerate eigenvalue

67. A child makes a random walk on a square lattice of lattice constant a taking a step in this north east, south or west directions with probabilities 0.255, 0.255, 0.245, and 0.245, respectively. After a large number of steps N, the expected position of the child with respect to the starting point is at a distance

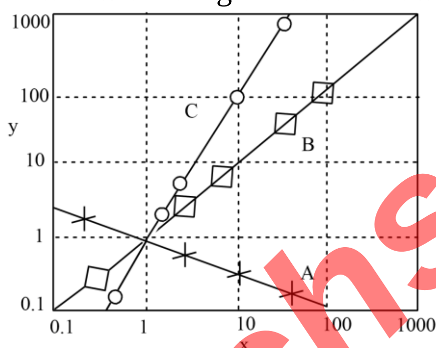
(a) $\sqrt{2}\times10^{-7}Na$ in the north-east direction (b) $\sqrt{2N}\times10^{-2}a$ in the north-east direction

(c) $2\sqrt{2}\times10^{-2}Na$ in the south-east direction (d) 0

68. A Carnot cycle operates as a heat engine between two bodies of equal heat capacity until their temperatures become equal. If the initial temperatures of the bodies are T_1 and T_2 respectively, and $T_1 > T_2$ then their common final temperature is

(a) T_1^2/T_2 (b) T_2^2/T_1 (c) $\sqrt{T_1T_2}$ (d) $\frac{1}{2}(T_1+T_2)$

69. Three sets of data A, B and C from an experiment, represented by $\times$, □ and O, are plotted on log-log scale. Each of these are fitted with straight lines as shown in the figure.



the functional dependency $y(x)$ for the sets A, B and C are respectively

(a) $\sqrt{x}$, x and x^2 (b) $-\frac{x}{2}$, x and $2x$

(b) $\frac{1}{x^2}$, x and x^2 (d) $\frac{1}{\sqrt{x}}$, x and x^2

70. A sample of Si has electron an hole mobilities of 0.13 and 0.05 $m^2/V-s$ respectively at 300 K. It is doped with P and Al with doping densities of $1.5\times10^{21}/m^3$ and $2.5\times10^{21}/m^3$ respectively. The conductivity of the doped Si sample at 300 K is

(a) $8\,\Omega^{-1}m^{-1}$ (b) $32\,\Omega^{-1}m^{-1}$ (c) $20.8\Omega^{-1}m^{-1}$ (d) $83.2\Omega^{-1}m^{-1}$

71. A 4-variable switching function is given by $f=\Sigma(5,7,8,10,13,15)+d(0,1,2)$, where d is the do-not care condition. The minimized form of f in sum of products (SOP) form is

(a) $\bar{A}\bar{C}+\bar{B}\bar{D}$ (b) $A\bar{B}+C\bar{D}$ (c) $AD+BC$ (d) $\bar{B}\bar{D}+BD$

72. A perturbation $V_{pert}=aL^2$ is added to the Hydrogen atom potential. The shift in the energy level of the 2P state. When the effects of spin are neglected up to second order in a is

(a) 0 (b) $2a\hbar^2+a^2\hbar^4$ (c) $2a\hbar^2$ (d) $a\hbar^2+\frac{3}{2}a^2\hbar^4$

73. A gas laser cavity has been designed to operate at $\lambda = 0.5\mu m$ with a cavity length of 1m. with this set-up, the frequency is fund to be larger than the desired frequency by 100 Hz. The change in the effective length of the cavity required to retune the laser is

(a) $-0.334\times10^{-12}m$ (b) $0.334\times10^{-12}m$

(c) $0.167\times10^{-12}m$ (d) $-0.167\times10^{-12}m$

74. The spectroscopic symbol for the ground state of $_{31}$Al is $^2P_{1/2}$. Under the action of a strong magnetic field (when L-S coupling can be neglected) the ground state energy level will split into

(a) 3 levels (b) 4 levels (c) 5 levels (d) 6 levels

75. A uniform linear monoatomic chain is modelled by a spin-mass system m separated by nearest neighbour distance a and spring constant $m\omega_0^2$. The dispersion relation for this system is

(a) $\omega(k) = 2\omega_0\left(1-\cos\left(\frac{ka}{2}\right)\right)$ (b) $\omega(k) = 2\omega_0\sin^2\left(\frac{ka}{2}\right)$

(c) $\omega(k) = 2\omega_0\sin\left(\frac{ka}{2}\right)$ (d) $\omega(k) = 2\omega_0\tan\left(\frac{ka}{2}\right)$