LEADING INSTITUTE FOR CSIR-JRF/NET, GATE & JAM

CSIR-UGC-NET/JRF DEC-2015

PHYSICAL SCIENCES DEC 2015

PART–B

21. In the scattering of some elementary particle, the scattering cross-section σ is found to depend on the total energy E and the fundamental constants h (Planck's constant) and c (the speed of light in vacuum). Using dimensional analysis, the dependence of σ on these quantities is given by

(a) $\sqrt{\frac{hc}{E}}$ (b) $\frac{hc}{E^{3/2}}$ (c) $\left(\frac{hc}{E}\right)^2$ (d) $\frac{hc}{E}$

22. If $y = \frac{1}{\tanh(x)}$, then x is:

(a) $\ln\left(\frac{y+1}{y-1}\right)$ (b) $\ln\left(\frac{y-1}{y+1}\right)$ (c) $\ln\sqrt{\frac{y-1}{y+1}}$ (d) $\ln\sqrt{\frac{y+1}{y-1}}$

23. The function $\frac{z}{sin\pi z^2}$ of a complex variable z has

(a) a simple pole at 0 and poles of order 2 at $\pm\sqrt{n}$ for $n = 1,2,3 \ldots$

(b) a simple pole at 0 and poles of order 2 at $\pm\sqrt{n}$ and $\pm i\sqrt{n}$ for $n = 1,2,3 \ldots$

(c) poles of order 2 at $\pm\sqrt{n}, n = 0,1,2,3 \ldots$

(d) poles of order 2 at $\pm n, n = 0,1,2,3 \ldots$

24. The Fourier transform of $f(x)$ is

$\bar{f} = \int_{-\infty}^{+\infty} dx\, e^{ikx} f(x)$. If

$f(x) = \alpha\delta(x) + \beta\delta'(x) + \gamma\delta''(x)$,

where $\delta(x)$ is the Dirac delta-function (and prime denotes derivative), what is $\bar{f}(k)$?

(a) $\alpha + i\beta k + i\gamma k^2$ (b) $\alpha + \beta k - \gamma k^2$

(c) $\alpha - i\beta k - \gamma k^2$ (d) $i\alpha + \beta k - i\gamma k^2$

25. The solution of the differential equation $\frac{dx}{dt} = 2\sqrt{1-x^2}$, with initial condition $x = 0$ at $t = 0$ is

(a) $x = \begin{cases} sin2t, 0 \le t < \frac{\pi}{4} \\ sinh2t, \quad t \ge \frac{\pi}{4} \end{cases}$

(b) $x = \begin{cases} sin2t, 0 \le t < \frac{\pi}{2} \\ 1, \quad t \ge \frac{\pi}{4} \end{cases}$

(c) $x = \begin{cases} sin2t, 0 \le t < \frac{\pi}{4} \\ 1, \quad t \ge \frac{\pi}{4} \end{cases}$

(d) $x = 1 - cos2t,\ t \ge 0$

26. A particle moves in three-dimensional space in a central potential $V(r) = kr^4$, where k is a constant. The angular frequency ω for a circular orbit depends on its radius R as

(a) $\omega \propto R$ (b) $\omega \propto R^{-1}$ (c) $\omega \propto R^{1/4}$ (d) $\omega \propto R^{-2/3}$

27. Two masses, m each, are placed at the points $(x, y) = (a, a)$ and $(-a, -a)$. Two masses, $2m$ each, are placed at the points $(a, -a)$ and $(-a, a)$. The principal moments of inertia of the system are

(a) $2ma^2, 4ma^2$ (b) $4ma^2, 8ma^2$ (c) $4ma^2, 4ma^2$ (d) $8ma^2, 8ma^2$

28. The Lagrangian of a system is given by

$$L = \frac{1}{2}m\dot{q}_1^2 + 2m\dot{q}_2^2 - k\left(\frac{5}{4}q_1^2 + 2q_2^2 - 2q_1q_2\right)$$

Where m and k are positive constants, the frequency of its normal modes are

(a) $\sqrt{\frac{k}{2m}}, \sqrt{\frac{3k}{m}}$

(b) $\sqrt{\frac{k}{2m}}, (13 \pm \sqrt{73})$

(c) $\sqrt{\frac{5k}{2m}}, \sqrt{\frac{k}{m}}$

(d) $\sqrt{\frac{k}{2m}}, \sqrt{\frac{6m}{m}}$

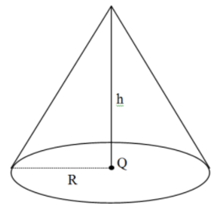
29. Consider a particle of mass m moving with a speed v. If T_R denotes the relativistic kinetic energy and T_N its non-relativistic approximation, then the value of $(T_R - T_N)/T_R$ for $v = 0.01c$, is

(a) 1.25×10^{-5} (b) 5.0×10^{-5} (c) 7.5×10^{-5} (d) 1.0×10^{-4}

30. A hollow metallic sphere of radius a, which is kept at a potential V_0, has a charge Q at its centre. The potential at a point outside the sphere, at a distance r from the centre, is

(a) V_0 (b) $\frac{Q}{4\pi\varepsilon_0 r} + \frac{V_0 a}{r}$ (c) $\frac{Q}{4\pi\varepsilon_0 r} + \frac{V_0 a^2}{r^2}$ (d) $\frac{V_0 a}{r}$

31. Consider a charge Q at the origin of 3-dimensional coordinate system. The flux of the electric field through the curved surface of a cone that has a height h and a circular base of radius R (as shown in the figure) is



(a) $\frac{Q}{\varepsilon_0}$ (b) $\frac{Q}{2\varepsilon_0}$ (c) $\frac{hQ}{R\varepsilon_0}$ (d) $\frac{QR}{2h\varepsilon_0}$

32. Given a uniform magnetic field $B = B_0\hat{k}$ (where B_0 is a constant), a possible choice for the magnetic vector potential A is

(a) $B_0 y\hat{\imath}$ (b) $-B_0 y\hat{\imath}$ (c) $B_0(x\hat{\jmath} + y\hat{\imath})$ (d) $B_0(x\hat{\jmath} - y\hat{\imath})$

33. A beam of unpolarized light in a medium with dielectric constant $\in_1$ is reflected from a plane interface formed with another medium of dielectric constant $\in_2 = 3\in_1$. The two media have identical magnetic permeability. If the angle of incident is 60^o, then the reflected light

(a) is plane polarized perpendicular to the plane of incidence

(b) is plane polarized parallel to the plane of incidence

(c) is circularly polarized

(d) has the same polarization as the incident light

34. A Hermitian operator $\hat{O}$ has two normalised eigenstates $|1\rangle$ and $|2\rangle$ with eigenvalues 1 and 2, respectively. The two states $|u\rangle = \cos\theta|1\rangle + \sin\theta|2\rangle$ and $|v\rangle = \cos\phi|1\rangle + \sin\phi|2\rangle$ are such that $\langle v|\hat{O}|v\rangle = 7/4$ and $\langle u|v\rangle = 0$. Which of the following are possible values of θ and ϕ ?

(a) $\theta = -\frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$ (b) $\theta = \frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$

(c) $\theta = -\frac{\pi}{4}$ and $\phi = \frac{\pi}{4}$ (d) $\theta = \frac{\pi}{3}$ and $\phi = -\frac{\pi}{6}$

35. The ground state energy of a particle of mass m in the potential $V(x) = V_0 cosh\left(\frac{x}{L}\right)$, where L and V_0 are constants (and $V_0 \gg \frac{\hbar^2}{2mL^2}$) is approximately

(a) $V_0 + \frac{\hbar}{L}\sqrt{\frac{2V_0}{m}}$ (b) $V_0 + \frac{\hbar}{L}\sqrt{\frac{V_0}{m}}$

(c) $V_0 + \frac{\hbar}{4L}\sqrt{\frac{V_0}{m}}$ (d) $V_0 + \frac{\hbar}{2L}\sqrt{\frac{V_0}{m}}$

36. Let ψ_{nlm} denote the eigenstates of a hydrogen atom in the usual notation. The state

$\frac{1}{5}[2\psi_{200} - 3\psi_{211} + \sqrt{7}\psi_{210} - \sqrt{5}\psi_{21-1}]$

is an eigenstate of

(a) L^2, but not of the Hamiltonian of L_z

(b) the Hamiltonian, but not of L^2 or L_z

(c) the Hamiltonian, L^2 and L_z

(d) L^2 and L_z but not of the Hamiltonian

37. The Hamiltonian for a spin-1/2 particle at rest is given by $H = E_0(\sigma_z + \alpha\sigma_x)$, where σ_x and σ_z are Pauli spin matrices and E_0 and α are constants. The eigenvalues of this Hamiltonian are

(a) $\pm E_0\sqrt{1+\alpha^2}$ (b) $\pm E_0\sqrt{1-\alpha^2}$

(c) E_0 (doubly degenerate) (d) $E_0\left(1 \pm \frac{1}{2}\alpha^2\right)$

38. The heat capacity of (the interior of) a refrigerator is 4.2 kJ/K. The minimum work that must be done to lower the internal temperature from 18°C to $17^\circ C$ when the outside temperature is $27^\circ C$ is

(a) 2.20 kJ (b) 0.80 kJ (c) 0.30 kJ (d) 0.14 kJ

39. For a system of independent non-interacting one-dimensional oscillators, the value of the free energy per oscillator, in the limit $T \to 0$, is

(a) $\frac{1}{2}\hbar\omega$ (b) $\hbar\omega$ (c) $\frac{3}{2}\hbar\omega$ (d) 0

40. The partition function of a system of N Ising spins is $Z = \lambda_1^N + \lambda_2^N$, where λ_1 and λ_2 are functions of temperature, but are independent of N. If $\lambda_1 > \lambda_2$, the free energy per spin in the limit $N \to \infty$ is

(a) $-k_B T \ln\left(\frac{\lambda_1}{\lambda_2}\right)$ (b) $-k_B T \ln \lambda_2$ (c) $-k_B T \ln(\lambda_1\lambda_2)$ (d) $-k_B T \ln \lambda_1$

41. The Hamiltonian of a system of N non interacting spin-1/2 particles is $H = -\mu_0 B \Sigma_i S_i^z$, where $S_i^z = \pm 1$ are the component of i^{th} spin along an external magnetic field B. At a temperature T such that $e^{\mu_0 B/k_B T} = 2$, the specific heat per particle is

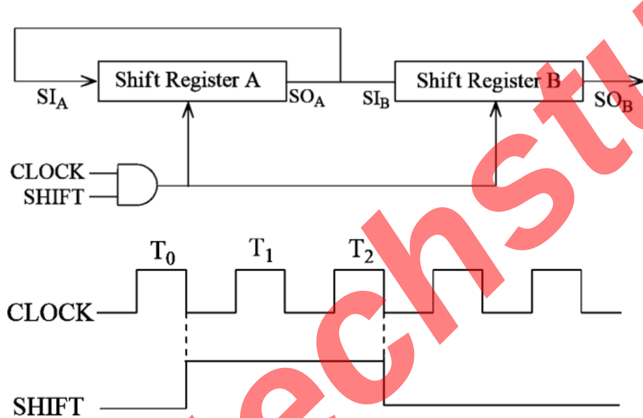
(a) $\frac{16}{25} k_B$ (b) $\frac{8}{25} k_B \ln 2$ (c) $k_B (\ln 2)^2$ (d) $\frac{16}{25} k_B (\ln 2)^2$

42. If the reverse bias voltage of a silicon varactor is increased by a factor of 2, the corresponding transition capacitance

(a) increases by a factor of $\sqrt{2}$ (b) increases by a factor of 2

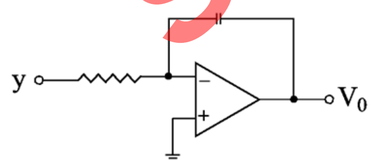
(c) decreases by a factor of $\sqrt{2}$ (d) decreases by a factor of 2

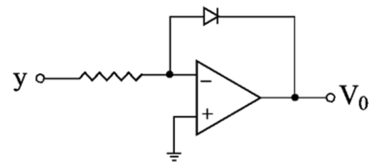
43. In the schematic figure below, the initial values of 4bit shift registers A and B are 1011 and 0010 respectively. The values at SO_A and SO_B after the pulse T_2 are respectively

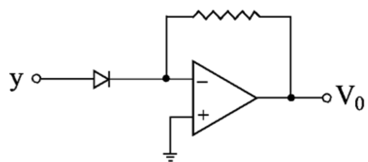


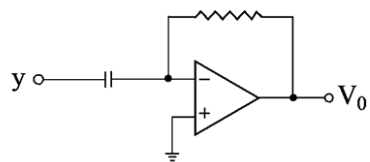
(a) 1110 and 1001 (b) 1101 and 1001
(c) 1101 and 1100 (d) 1110 and 1100

44. If the parameters y and x are related by $y = \log(x)$, then the circuit that can be used to produce an output voltage V_0 varying linearly with x is

(a) 

(b) 

(c) 

(d) 

45. Two data sets A and B consist of 60 and 10 readings of a voltage measured using voltmeters of resolution of $1mV$ and $0.5\ mV$ respectively. The uncertainly in the mean voltage obtained from the data sets A and B are U_A and U_B, respectively. If the uncertainity of mean of the combined data sets is U_{AB}, then which of the following statements is correct?

(a) $U_{AB} < U_A$ and $U_{AB} > U_B$ (b) $U_{AB} < U_A$ and $U_{AB} < U_B$

(c) $U_{AB} > U_A$ and $U_{AB} < U_B$ (d) $U_{AB} > U_A$ and $U_{AB} > U_B$

46. The Hermit polynomial $H_n(x)$ satisfies the differential equation

$$\frac{d^2H_n}{dx^2} - 2x\frac{dH_n}{dx} + 2nH_n(x) = 0$$

The corresponding generating function $G(t,x) = \Sigma_{n=0}^{\infty}\frac{1}{n!}H_n(x)t^n$ satisfies the equation

(a) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2t\frac{\partial G}{\partial t} = 0$ (b) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} - 2t^2\frac{\partial G}{\partial t} = 0$

(c) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2\frac{\partial G}{\partial t} = 0$ (d) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2\frac{\partial^2 G}{\partial x \partial t} = 0$

47. A function $f(x)$ satisfies the differential equation $\frac{d^2 f}{dx^2} - \omega^2 f = -\delta(x-a)$, where ω is positive. The Fourier transforms $\tilde{f}(k) = \int_{-\infty}^{+\infty} dx\, e^{ikx} f(x)$ of f, and the solution of the equation are respectively,

(a) $\frac{e^{ika}}{k^2+\omega^2}$ and $\frac{1}{2\omega}\left(e^{-\omega|x-a|} + e^{\omega|x-a|}\right)$ (b) $\frac{e^{ika}}{k^2+\omega^2}$ and $\frac{1}{2\omega}e^{-\omega|x-a|}$

(c) $\frac{e^{ika}}{k^2-\omega^2}$ and $\frac{1}{2\omega}\left(e^{-i\omega|x-a|} + e^{i\omega|x-a|}\right)$ (d) $\frac{e^{ika}}{k^2-\omega^2}$ and $\frac{1}{2i\omega}\left(e^{-i\omega|x-a|} - e^{i\omega|x-a|}\right)$

48. For a dynamical system by the equation $\frac{dx}{dt} = 2\sqrt{1-x^2}$, with $|x| \le 1$,

(a) $x = -1$ and $x = 1$ are both unstable fixed points

(b) $x = -1$ and $x = 1$ are both stable fixed points

(c) $x = -1$ is an unstable fixed point and $x = 1$ is a stable fixed point

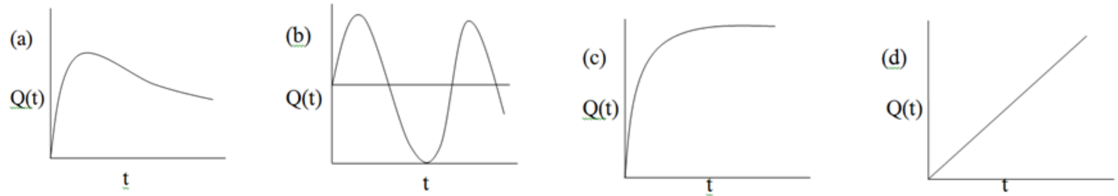
(d) $x = -1$ is a stable fixed point and $x = 1$ is an unstable fixed point

49. The value of the integral $\int_0^8 \frac{1}{x^2+5}dx$, evaluated using Simpson's $\frac{1}{3}$ rule with $h = 2$, is

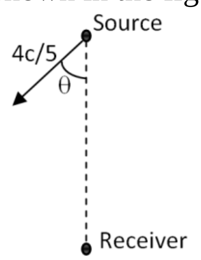
(a) 0.565 (b) 0.620 (c) 0.698 (d) 0.736

50. A canonical transformation $(p, q) \rightarrow (P, Q)$is performed on the Hamiltonian

$H = \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2 q^2$ Vi a the generating function $F = \frac{1}{2}m\omega q^2 cotQ$. If $Q(0) = 0$, which of the following graphs shows schematically the dependence of $Q(t)$ on t?



51. A distant source, emitting radiation of frequency ω, moves with a velocity $4c/5$ in a certain direction with respect to a receiver (as shown in the figure)



The upper cut-off frequency of the receiver is $3\omega/2$. Let θ be the angle as shown. For the receiver to detect the radiation θ should at least be

(a) $\cos^{-1}\left(\frac{1}{2}\right)$ (b) $\cos^{-1}\left(\frac{3}{4}\right)$ (c) $\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (d) $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

52. The Lagrangian of a particle moving in a plane is given in Cartesian coordinates as

$$L = \dot{x}\dot{y} - x^2 - y^2$$

In polar coordinates the expression for the canonical momentum p_r (conjugate to the radial coordinate r) is

(a) $\dot{r}sin\theta + r\dot{\theta}cos\theta$ (b) $\dot{r}cos\theta + r\dot{\theta}sin\theta$
(c) $2\dot{r}cos2\theta - r\dot{\theta}sin2\theta$ (d) $\dot{r}sin2\theta + r\dot{\theta}cos2\theta$

53. A small magnetic needle is kept at (0, 0) with its moment along the x-axis. Another small magnetic needle is at the point (1, 1) and is free to rotate in the xy-plane. In equilibrium the angle θ between their magnetic moments is such that

(a) $\tan\theta = 1/3$ (b) $\tan\theta = 0$ (c) $\tan\theta = 3$ (d) $\tan\theta = 1$

54. A dipole of moment $\vec{p}$, oscillating at frequency ω, radiates spherical waves. The vector potential at large distance is

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} i\omega \frac{e^{ikr}}{r}\vec{p}.$$

To order $(1/r)$the magnetic field $\vec{B}$at a point $\vec{r} = r\hat{n}$ is

(a) $-\frac{\mu_0}{4\pi}\frac{\omega^2}{c}(\hat{n}\cdot\vec{p})\vec{n}\frac{e^{ikr}}{r}$ (b) $-\frac{\mu_0}{4\pi}\frac{\omega^2}{c}(\hat{n}\times\vec{p})\frac{e^{ikr}}{r}$

(c) $-\frac{\mu_0}{4\pi}\omega^2 k(\hat{n}\cdot\vec{p})\vec{p}\frac{e^{ikr}}{r}$ (d) $-\frac{\pi_0}{4\pi}\frac{\omega^2}{c}\vec{p}\frac{e^{ikr}}{r}$

55. The frequency dependent dielectric constant of a material is given by

$$\varepsilon(\omega) = 1 + \frac{A}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

where A is a positive constant, ω_0 the resonant frequency and γ the damping coefficient. For an electromagnetic wave of angular frequency $\omega \ll \omega_0$. Which of the following is true? (Assume that $\frac{\gamma}{\omega_0} \ll 1$)

(a) there is negligible absorption of the wave
(b) the wave propagation is highly dispersive
(c) there is strong absorption of the electromagnetic wave
(d) the group velocity and the phase velocity will have opposite sign

56. A hydrogen atom is subjected to the perturbation $V_{pert}(r) = \epsilon\cos(2r/a_0)$,

where a_0 is the Bohr radius. The change in the ground state energy to first order in ϵ is

(a) $\epsilon/4$ (b) $\epsilon/2$ (c) $-\epsilon/2$ (d) $-\epsilon/4$

57. A positron is suddenly absorbed by the nucleus of a tritium ($^3_1$H)atom to turn the latter into a He^+ion. If the electron in the tritium atom was initially in the ground state, the probability that the resulting He^+ion will be in its ground state is

(a) 1 (b) $\frac{8}{9}$ (c) $\frac{128}{243}$ (d) $\frac{512}{729}$

58. The product of the uncertainties $(\Delta L_x)(\Delta L_y)$for a particle in the state $a|1,1\rangle + b1,-1\rangle$ (where $|l,m\rangle$ denotes an eigenstates of L^2 and L_z) will be a minimum for

(a) $a = \pm ib$ (b) $a = 0$ and $b = 1$ (c) $a = \frac{\sqrt{3}}{2}$ and $b = \frac{1}{2}$ (d) $a = \pm b$

59. The ground state energy of a particle in the potential $V(x) = g|x|$, estimated using the trial wavefunction

$$\psi(x) = \begin{cases} \sqrt{\frac{c}{a^2}}(a^2 - x^2), & x < |a| \\ 0 \quad , & x \geq |a| \end{cases}$$

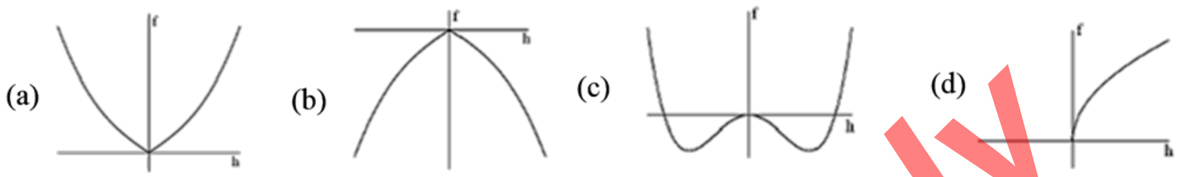
(where g and c are constants) is

(a) $\frac{15}{16}\left(\frac{\hbar^2 g^2}{m}\right)^{1/3}$ (b) $\frac{5}{6}\left(\frac{\hbar^2 g^2}{m}\right)^{1/3}$ (c) $\frac{3}{4}\left(\frac{\hbar^2 g^2}{m}\right)^{1/3}$ (d) $\frac{7}{8}\left(\frac{\hbar^2 g^2}{m}\right)^{1/3}$

60. An ensemble of non-interacting spin-1/2 particles is in contact with a heat bath at temperature T, and is subjected to an external magnetic field. Each particle can be in one of the two quantum states of energies $\pm\epsilon_0$. If the mean energy per particle is $-\epsilon_0/2$, then the free energy per particle is

(a) $-2\epsilon_0 \frac{\ln\left(\frac{4}{\sqrt{3}}\right)}{\ln 3}$ (b) $-\epsilon_0 \ln\left(\frac{3}{2}\right)$ (c) $-2\epsilon_0 \ln 2$ (d) $-\epsilon_0 \frac{\ln 2}{\ln 3}$

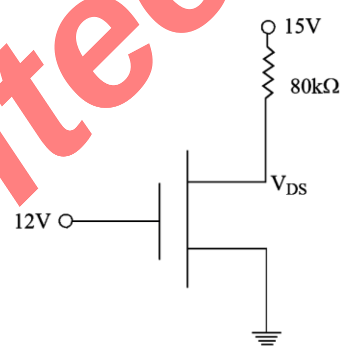
61. Which of the following graphs shows the qualitative dependence of the free energy $f(h,T)$ of a ferromagnet in an temperature $T < T_c$ where T_c is the critical temperature



62. Consider a random walker on a square lattice at each step the walker moves to a nearest neighbour site with equal probability for each of the four sites. The walker starts at the origin and takes 3 steps. The probability that during this walk no site is visited more than once is

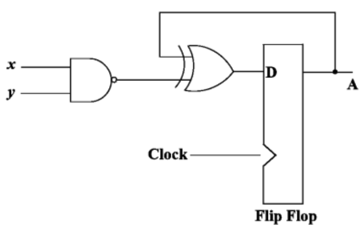
(a) $12/27$ (b) $27/64$ (c) $3/8$ (d) $9/16$

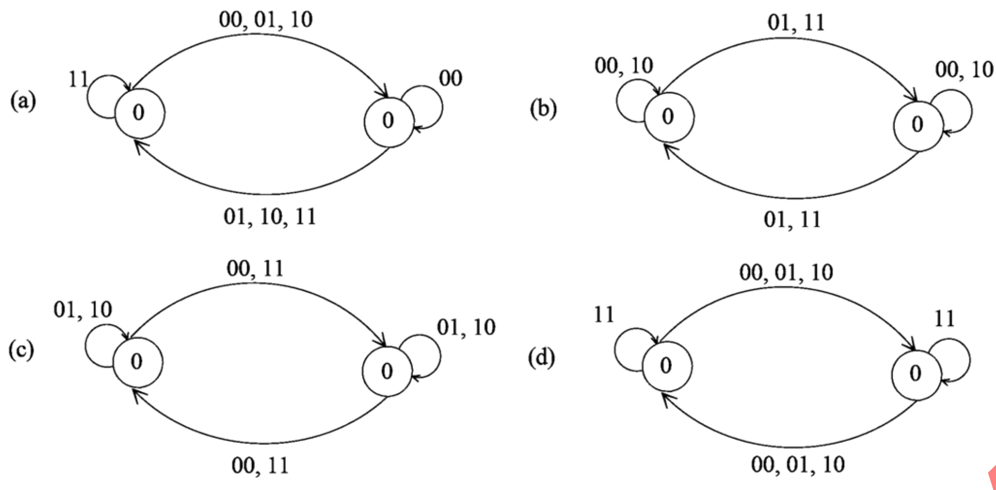
63. Consider an n-MOSFET with the following parameters: current drive strength $K = 60\mu A/V^2$, breakdown voltage $BV_{DS} = 10V$, ratio of effective gate width to the channel length $\frac{W}{L} = 5$ and threshold voltage $V_{th} = 0.5V$. In the circuit given below, this n-MOSFET is operating in the



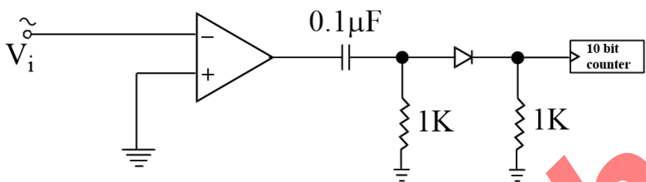
(a) ohmic region (b) cut-off region
(c) saturation region (d) breakdown region

64. The state diagram corresponding to the following circuit is





65. A sinusoidal signal of peak to peak amplitude $1V$ and unknown time period is input to the following circuit for 5 seconds duration. If the counter measures a value $(3E8)_H$ in hexadecimal then the time period of the input signal is



(a) 2.5 *ms* (b) 4 *ms* (c) 10 *ms* (d) 5 *ms*

66. The first order diffraction peak of a crystalline solid occurs at a scattering angle of 30° when the diffraction pattern is recorded using an X-ray beam of wavelength 0.15 *nm*. If the error in measurements of the wavelength and the angel are 0.01 *nm* and 1° respectively, then the error in calculating the inter planar spacing will approximately be.

(a) 1.1×10^{-2} nm (b) 1.3×10^{-4} nm (c) 2.5×10^{-2} nm (d) 2.0×10^{-3} nm

67. The dispersion relation of electrons in a 3-dimensinal lattice in the tight binding approximation is given by,

$$\varepsilon_k = \alpha cos k_x a + \beta cos k_y a + \gamma cos k_z a$$

where a is the lattice constant and α, β, γ are constants with dimension of energy. The effective mass tensor at the corner of the first Brillouin zone $\left(\frac{\pi}{a}, \frac{\pi}{a}, \frac{\pi}{a}\right)$ is

(a) $\frac{\hbar^2}{a^2}\begin{pmatrix} -\frac{1}{\alpha} & 0 & 0 \\ 0 & -\frac{1}{\beta} & 0 \\ 0 & 0 & \frac{1}{\gamma} \end{pmatrix}$ (b) $\frac{\hbar^2}{a^2}\begin{pmatrix} -\frac{1}{\alpha} & 0 & 0 \\ 0 & -\frac{1}{\alpha} & 0 \\ 0 & 0 & -\frac{1}{\gamma} \end{pmatrix}$

(c) $\frac{\hbar^2}{a^2}\begin{pmatrix} \frac{1}{\alpha} & 0 & 0 \\ 0 & \frac{1}{\beta} & 0 \\ 0 & 0 & \frac{1}{\gamma} \end{pmatrix}$

(d) $\frac{\hbar^2}{a^2}\begin{pmatrix} \frac{1}{\alpha} & 0 & 0 \\ 0 & \frac{1}{\beta} & 0 \\ 0 & 0 & -\frac{1}{\gamma} \end{pmatrix}$

68. A thin metal film of dimension $2\text{mm}\times 2\text{mm}$ contains 4×10^{12} electrons. The magnitude of the Fermi wave vector of the system, in the free electron approximation is

(a) $2\sqrt{\pi}\times 10^7\,\text{cm}^{-1}$

(b) $\sqrt{2\pi}\times 10^7\,\text{cm}^{-1}$

(c) $\sqrt{\pi}\times 10^7\,\text{cm}^{-1}$

(d) $2\pi\times 10^7\,\text{cm}^{-1}$

69. For an electron moving through a one-dimensional periodic lattice of periodicity a, which of the following corresponding to an energy eigen function consistent with Bloch's theorem?

(a) $\psi(x) = A\,exp\left(i\left[\frac{\pi x}{a} + cos\left(\frac{\pi x}{2a}\right)\right]\right)$

(b) $\psi(x) = A\,exp\left(i\left[\frac{\pi x}{a} + cos\left(\frac{2\pi x}{2a}\right)\right]\right)$

(c) $\psi(x) = A\,exp\left(i\left[\frac{2\pi x}{a} + i\,cos\left(\frac{2\pi x}{2a}\right)\right]\right)$

(d) $\psi(x) = A\,exp\left(i\left[\frac{\pi x}{2a} + i\left|\frac{\pi x}{2a}\right|\right]\right)$

70. The LS configuration of the ground state of ^{12}Mg, ^{13}Al, ^{17}Cl and ^{18}Ar are respectively

(a) $^3S_1$, $^2P_{1/2}$, $^2P_{1/2}$ and $^1S_0$

(b) $^3S_1$, $^2P_{3/2}$, $^2P_{3/2}$ and $^3S_0$

(c) $^1S_0$, $^2P_{1/2}$, $^2P_{3/2}$ and $^1S_0$

(d) $^1S_0$, $^2P_{3/2}$, $^2P_{1/2}$ and $^3S_1$

71. For a two level system, the population of atoms in the upper and lower levels are 3×10^{18} and 0.7×10^{18}, respectively. If the coefficient of stimulated emission is $3.0\times 10^5\,m^3/W-s^3$ and the energy density is $9.0\,J/m^3-Hz$, the rate of stimulated emission will be

(a) $6.3\times 10^{16}\,s^{-1}$

(b) $4.1\times 10^{16}\,s^{-1}$

(c) $2.7\times 10^{16}\,s^{-1}$

(d) $1.8\times 10^{16}\,s^{-1}$

72. The first ionization potential of K is 4.34 eV, the electron affinity of Cl is 3.82eV and the equilibrium separation of KCl is 0.3 nm. The energy required to dissociate a KCl molecule into a K and a Cl atom is

(a) 8.62 eV (b) 8.16 eV (c) 4.28 eV (d) 4.14 eV

73. Consider the following processes involving free particles

(i) $\bar{n} \rightarrow \bar{p} + e^+ + \bar{\nu}_e$

(ii) $\bar{p} + n \rightarrow \pi^-$

(iii) $p + n \rightarrow \pi^+ + \pi^0 + \pi^0$

(iv) $p + \bar{\nu}_e \rightarrow n + e^+$

Which of the following statements is true?

(a) process (i) obeys all conservation laws.
(b) process (ii) conserves baryon number but violates energy-momentum conservation.
(c) process (iii) is not allowed by strong interactions, but is allowed by weak interaction.
(d) process (iv) conserves baryon number, but violates lepton number conservation.

74. The electric quadrupole moment of an odd proton nucleus is $\frac{(2j-1)}{2(j+1)}\langle r^2\rangle$, where j is the total angular momentum. Given that $R_0 = 1.2fm$, what is the value, in barn, of the quadrupole moment of the ^{27}Al nucleus in the shell model?

(a) 0.043 (b) 0.023 (c) 0.915 (d) 0

75. Of the nuclei of mass number $A = 125$, the binding energy calculated from the liquid drop model (given that the coefficients or the Coulomb and the asymmetry energy are $a_c = 0.7MeV$ and $a_{sym} = 22.5MeV$ respectively) is a maximum for

(a) $^{125}_{54}Xe$ (b) $^{125}_{53}I$ (c) $^{125}_{52}Te$ (d) $^{125}_{51}Sb$