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LEADING INSTITUE FOR CSIR-JRF/NET, GATE & JAM

CSIR-UGC-NET/JRF JUNE-2013

PHYSICAL SCIENCES JUNE 2013

PART–B

21. Two identical bosons of mass m are placed in a one dimensional potential $V(x)=\frac{1}{2}m\omega^2x^2$. The bosons interact via a weak potential

$$V_{12}=V_0\exp\left(\frac{-m\Omega(x_1-x_2)^2}{4\hbar}\right)$$

where, x_1 and x_2 denote coordinates of the particles. Given that the ground state wave-function of the harmonic oscillator is $\Psi_0(x)=\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}e^{-\frac{m\omega x^2}{2\hbar}}$.

The ground state energy of the two boson system, to the first order in V_0 is

(a) $\hbar\omega+2V_0$
(b) $\hbar\omega+\frac{V_0\Omega}{\omega}$
(c) $\hbar\omega+V_0\left(1+\frac{\Omega}{2\omega}\right)^{-1/2}$
(d) $\hbar\omega+V_0\left(1+\frac{\omega}{\Omega}\right)$

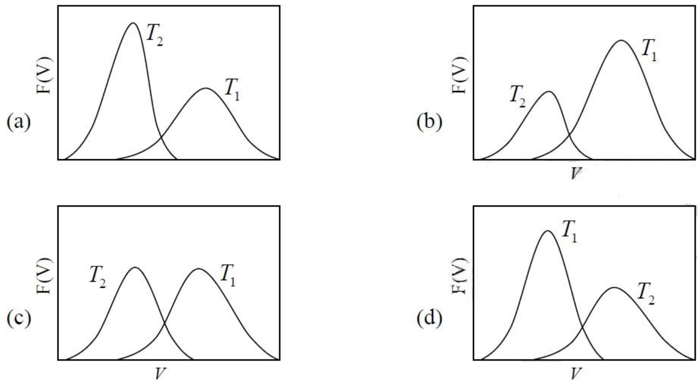
22. Ten grams of ice at 0^oC is added to a beaker containing 30 grams of water at 25^oC. What is the final temperature of the system when it comes to thermal equilibrium? (The specific heat of water is 1cal/gm/^{o}C and latent heat of melting of ice is 80 cal/gm.)

(a) 0^oC
(b) 7.5^oC
(c) 12.5^oC
(d) -12.5^oC

23. A vessel has two compartments of volume V_1 and V_2, containing an ideal gas at pressures p_1 and p_2 and temperatures T_1 and T_2 respectively. If the wall separating the compartments is removed, the resulting equilibrium temperature will be:

(a) $\frac{p_1T_1+p_2T_2}{p_1+p_2}$
(b) $\frac{V_1T_1+V_2T_2}{V_1+V_2}$
(c) $\frac{p_1V_1+p_2V_2}{(p_1V_1/T_1)+(p_2V_2/T_2)}$
(d) $(T_1T_2)^{1/2}$

24. For temperature $T_1 > T_2$, the qualitative temperature dependence of the probability distribution $F(v)$ of speed v of a molecule in three dimensions is correctly represented by the following figure:



25. A system of non-interacting spin-1/2 charged particles are placed in an external magnetic field. At low temperature *T*, the leading behavior of the excess energy above the ground state energy depends on *T* as: (*c* is n constant)

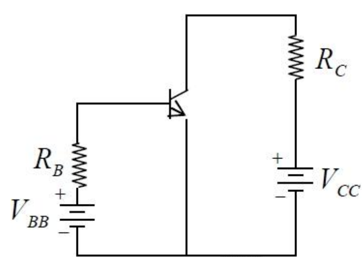
(a) cT (b) cT^3 (c) $e^{-c/T}$ (d) *c* (is dependent of *T*)

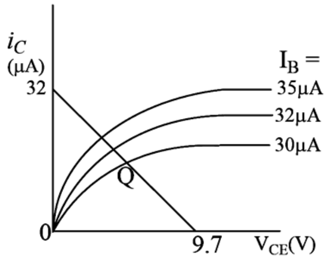
26. The acceleration due to gravity g is determined by measuring the time period *T* and the length *L* of a simple pendulum. If the uncertainties in the measurements of *T* and *L* are ΔT and ΔL respectively, the fractional error $\Delta g/g$ in measuring g is best approximated by

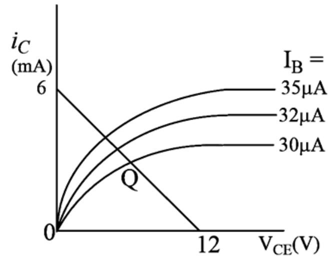
(a) $\frac{|\Delta L|}{L} + \frac{|\Delta T|}{T}$ (b) $\frac{|\Delta L|}{L} + \frac{|2\Delta T|}{T}$

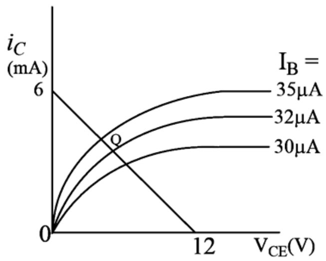
(c) $\sqrt{\frac{|\Delta L|^2}{L} + \frac{|\Delta T|^2}{T}}$ (d) $\sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{2\Delta T}{T}\right)^2}$

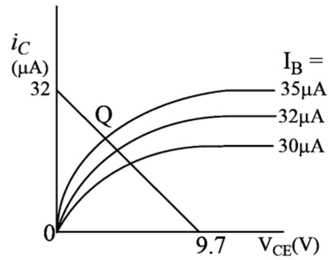
27. A silicon transistor with built in voltage 0.7 is used in the circuit shown, with $V_{BB} = 9.7V, R_n = 300k\Omega, V_{CC} = 12V$ and $R_C = 2k\Omega$. Which of the following figures correctly represents the load line and the quiescent *Q* point?



(a)


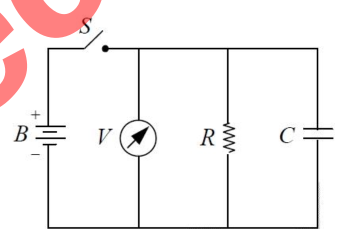
(b)


(c)


(d)


28. If the analog input to an 8-bit successive approximation ADC is increased from 1.0 V to 2.0 V, then the conversion time will
   (a) remain unchanged
   (b) double
   (c) decrease to half its original value
   (d) increase four times

29. The insulation resistance R of an insulated cable is measured by connecting it in parallel with a capacitor C, a voltmeter, and battery B as shown. The voltage across the cable dropped from 150V to 15V, 1000 seconds after the switch S is closed. If the capacitance of the cable is $5\mu F$, then its insulation resistance is approximately.

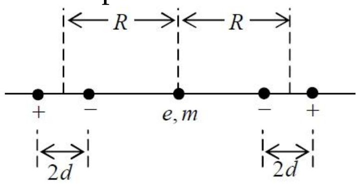


(a) $10^9\Omega$ (b) $10^8\Omega$ (c) $10^7\Omega$ (d) $10^6\Omega$

30. The approximation $\cos\theta \approx 1$ is valid up to 3 decimal places as long as $[\theta]$ is less than:
   (take $180^\circ/\pi = 57.29^\circ$)
   (a) 1.28° (b) 1.81° (c) 3.28° (d) 4.01°

31. The area of a disc in its rest frame S is equal to 1 (in some units). The disc will appear distorted to an observer O moving with a speed u with respect to S along the plane of the disc. The area of the disc measured in the rest frame of the observer O is (c is the speed of light in vaccum)
   (a) $\left(1-\frac{u^2}{c^2}\right)^{1/2}$ (b) $\left(1-\frac{u^2}{c^2}\right)^{-1/2}$ (c) $\left(1-\frac{u^2}{c^2}\right)$ (d) $\left(1-\frac{u^2}{c^2}\right)^{-1}$

32. A particle of charge e and mass m is located at the mid-point of the line joining two fixed collinear dipoles with unit charges as shown in the figure. (The particle is constrained to move only along the line joining the dipoles).Assuming that the length dipoles is much shorter than their separation, the natural frequency of oscillation of the particle is



(a) $\sqrt{\frac{6e^2R^2}{\pi\varepsilon_0 md^5}}$ (b) $\sqrt{\frac{6e^2R}{\pi\varepsilon_0 md^4}}$ (c) $\sqrt{\frac{6e^2d^2}{\pi\varepsilon_0 mR^5}}$ (d) $\sqrt{\frac{6e^2d}{\pi\varepsilon_0 mR^4}}$

33. A current I is created by a narrow beam of protons moving in vacuum with constant velocity $\vec{u}$. The direction and magnitude, respectively, of the pointing vector $\vec{S}$ outside the beam at a radial distance r (much larger than the width of the beam) from the axis, are

(a) $\vec{S}\perp\vec{u}$ and $|\vec{S}|=\frac{I^2}{4\pi^2\varepsilon_0|\vec{u}|r^2}$ (b) $\vec{S}\parallel(-\vec{u})$ and $|\vec{S}|=\frac{I^2}{4\pi^2\varepsilon_0|\vec{u}|r^4}$

(c) $\vec{S}\parallel\vec{u}$ and $|\vec{S}|=\frac{I^2}{4\pi^2\varepsilon_0|\vec{u}|r^3}$ (d) $\vec{S}\parallel\vec{u}$ and $|\vec{S}|=\frac{I^2}{4\pi^2\varepsilon_0|\vec{u}|r^4}$

34. If the electric and magnetic fields are unchanged when the vector potential $\vec{A}$ changes (in suitable units) according to $\vec{A}\to\vec{A}+\hat{r}$, where $\vec{r}=r(t)\hat{r}$, then the scalar potential ϕ must simultaneously change to

(a) $\phi-r$ (b) $\phi+r$ (c) $\phi-\frac{\partial r}{\partial t}$ (d) $\phi+\frac{\partial r}{\partial t}$

35. Consider an axially symmetric static charge distribution of the form, $\rho=\rho_0\left(\frac{r_0}{r}\right)^2 e^{-r/r_0}\cos^2\varphi$. The radial component of the dipole moment due to this charge distribution is

(a) $2\pi\rho_0 r_0^4$ (b) $\pi\rho_0 r_0^4$ (c) $\rho_0 r_0^4$ (d) $\pi\rho_0 r_0^4/2$

36. In a basis in which the z-component S_z of the spin is diagonal, an electron is in a spin state

$$\Psi=\begin{pmatrix}(1+i)/\sqrt{6}\\ \sqrt{2/3}\end{pmatrix}$$

The probabilities that a measurement of S_z will yield the values $\hbar/2$ and $-\hbar/2$ are, respectively,

(a) $\frac{1}{2}$ and $\frac{1}{2}$ (b) $\frac{2}{3}$ and $\frac{1}{3}$ (c) $\frac{1}{4}$ and $\frac{3}{4}$ (d) $\frac{1}{3}$ and $\frac{2}{3}$

37. Consider the normalized state $|\Psi\rangle$ of a particle in a one-dimensional harmonic oscillator:

$|\Psi\rangle = b_1|0\rangle + b_2|1\rangle$

Where, $|0\rangle$ and $|1\rangle$ denote the ground and first excited states respectively, and b_1 and b_2 are real constants. The expectation value of the displacement x in the state $|\Psi\rangle$ will be a minimum when

(a) $b_2 = 0, b_1 = 1$ (b) $b_2 = \frac{1}{\sqrt{2}} b_1$ (c) $b_2 = \frac{1}{2} b_1$ (d) $b_2 = b_1$

38. A muon (μ^-) from cosmic rays is trapped by a proton to form a hydrogen like atom. Given that a muon is approximately 200 times heavier than an electron, the longest wavelength of the spectral line (in the analogue of the Lyman series) of such an atom will be:

(a) $5.62\dot{A}$ (b) $6.67\dot{A}$ (c) $3.75\dot{A}$ (d) $13.3\dot{A}$

39. The un-normalized wave-function of a particle in a spherically symmetric potential is given by $\Psi(\vec{r}) = 2f(r)$, where $f(r)$ is a function of the radial variable r. The eigen value of the operator $\vec{L}^2$ (namely the square of the orbital angular momentum) is

(a) $\frac{\hbar^2}{4}$ (b) $\frac{\hbar^2}{2}$ (c) $\hbar^2$ (d) $2\hbar^2$

40. Given that, $\sum_{n=0}^{\infty} H_n(x) \frac{t^n}{n!} = e^{-t^2+2tx}$; the value of $H_4(0)$ is

(a) 12 (b) 6 (c) 24 (d) –6

41. A unit vector $\hat{n}$ on the xy-plane is at an angle of 120^o with respect to i. The angle between the vectors $\vec{u} = a\hat{i} + b\hat{n}$ and $\vec{v} = a\hat{n} + b\hat{i}$ will be 60^o if

(a) $b = \sqrt{3}a/2$ (b) $b = 2a/\sqrt{3}$ (c) $b = a/2$ (d) $b = a$

42. With $z = x + iy$, which of the following functions $f(x, y)$ is not a (complex) analytic function of z ?

(a) $f(x, y) = (x + iy - 8)^3 (4 + x^2 - y^2 + 2ixy)^7$

(b) $f(x, y) = (x + iy)^7 (1 - x - iy)^2$

(c) $f(x, y) = \left(x^2 - y^2 + 2ixy - 3\right)^5$

(d) $f(x, y) = \left(1 - x + iy\right)^4 \left(2 + x + iy\right)^6$

43. A planet of mass m and an angular momentum L moves in a circular orbit in a potential, $V(r) = -k/r$ where k is a constant. If it is slightly perturbed radially, the angular frequency of radial oscillations is

(a) $mk^2/\sqrt{2}L^3$ (b) mk^2/L^3

(c) $\sqrt{2}\,mk^2/L^3$ (d) $\sqrt{3}\,mk^2/L^3$

44. The Lagrangian of a particle of mass m moving in one dimension is given by, $L=\frac{1}{2}m\dot{x}^2-bx$; where b is a positive constant. The coordinate of the particle $x(t)$ at time t is given by: (in the following c_1 and c_2 are constants)

(a) $-\frac{b}{2m}t^2+c_1t+c_2$ (b) c_1t+c_2

(c) $c_1\cos\left(\frac{bt}{m}\right)+c_2\sin\left(\frac{bt}{m}\right)$ (d) $c_1\cosh\left(\frac{bt}{m}\right)+c_2\sinh\left(\frac{bt}{m}\right)$

45. A uniform cylinder of radius r and length l, and a uniform sphere of radius R are released on an inclined plane when their centers of mass are at the same height. If they roll down without slipping, and if the sphere reaches the bottom of the plane with a speed V, then the speed of the cylinder when it reaches the bottom is

(a) $V\sqrt{\frac{14rl}{15R^2}}$ (b) $4V\sqrt{\frac{r}{15R}}$ (c) $\frac{4V}{\sqrt{15}}$ (d) $V\sqrt{\frac{14}{15}}$

46. The components of a vector potential $A_\mu=(A_0,A_1,A_2,A_3)$ are given by $A_\mu=k(-xyz,yzt,zxt,xyt)$, where k is a constant. The three components of the electric field are

(a) $k(yz,zx,xy)$ (b) $k(x,y,z)$

(c) $(0,0,0)$ (d) $k(xt,yt,zt)$

47. In the Born approximation, the scattering amplitude $f(\theta)$ for the Yukawa potential $V(r)=\frac{\beta e^{-\mu r}}{r}$ is given by: (in the following $b=2k\sin\frac{\theta}{2}, E=\frac{\hbar^2k^2}{2m}$)

(a) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)^2}$ (b) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)^2}$

(c) $-\frac{2m\beta}{\hbar^2\sqrt{\mu^2+b^2}}$ (d) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)^3}$

48. If Ψ_{nlm} denotes the eigen-function of the Hamiltonian with a potential $V=V(r)$, then the expectation value of the operator L_x^2 and L_y^2 in the state $\Psi=\frac{1}{5}\left(3\Psi_{211}+\Psi_{210}-\sqrt{5}\Psi_{21-1}\right)$is

(a) $39\hbar^2/25$ (b) $13\hbar^2/25$ (c) $2\hbar^2$ (d) $26\hbar^2/25$

49. An oscillating current $I(t)=I_0\exp(-i\omega t)$ flows in the direction of the y-axis through a thin metal sheet of area $1.0cm^2$ kept in the xy-plane. The rate of total energy radiated per unit area from the surfaces of the metal sheet at a distance of 100 m is:

(a) $I_0\omega/(12\pi\varepsilon_0c^3)$ (b) $I^2{}_0\omega^2/(12\pi\varepsilon_0c^3)$

(c) $I_0^2\omega/(12\pi\varepsilon_0c^3)$ (d) $I_0\omega^2/(24\pi\varepsilon_0c^3)$

50. Consider a two dimensional infinite square well $V(x,y)=\begin{cases}0 & 0<x<a,\quad 0<y<a\\ \infty & \text{otherwise}\end{cases}$.

Its normalized Eigen functions are $\Psi_{n_x,n_y}(x,y)=\frac{2}{a}\sin\left(\frac{n_x\pi x}{a}\right)\sin\left(\frac{n_y\pi y}{a}\right)$; where $n_x,n_y=1,2,3.....$

If a perturbation $H'=\begin{cases}V_0 & 0<x<\frac{a}{2},\quad 0<y<\frac{a}{2}\\ 0 & \text{otherwise}\end{cases}$ is applied, then the correction to the energy of the first excited state to order V_0 is

(a) $\frac{V_0}{4}$ (b) $\frac{V_0}{4}\left(1\pm\frac{64}{9\pi^2}\right)$

(c) $\frac{V_0}{4}\left(1\pm\frac{16}{9\pi^2}\right)$ (d) $\frac{V_0}{4}\left(1\pm\frac{32}{9\pi^2}\right)$

51. Consider a system of two Ising spins S_1 and S_2 taking values ±1 with interaction energy given by $\varepsilon=-JS_1S_2$, when it is in thermal equilibrium at temperature T. For large T, the average energy of the system varies as C/k_BT, with C given by

(a) $-2J^2$ (b) $-J^2$ (c) J^2 (d) $4J$

52. Consider three particles A, B and C each with an attribute S that can take two values ±1. Let $S_A=1, S_B=1$ and $S_C=-1$ at a given instant. In the next instant, each S value can change to $-S$ with probability $1/3$. The probability that $S_A+S_B+S_C$ remains unchanged is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{9}$ (d) $\frac{4}{9}$

53. He bound on the ground state energy of the Hamiltonian with an attractive delta-function potential namely $H=-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}-\alpha\delta(x)$ using the variational principle with the trial wave function $\Psi(x)=A\exp(-bx^2)$ is

[Note: $\int_0^\infty e^{-t}t^a dt=\Gamma(a+1)$]

(a) $-m\alpha^2/4\pi\hbar^2$ (b) $-m\alpha^2/2\pi\hbar^2$

(c) $-m\alpha^2/\pi\hbar^2$ (d) $-m\alpha^2/\sqrt{5}\pi\hbar^2$

54. Consider two different systems each with three identical non-interacting particles. Both have single particle states with energies $\varepsilon_0, 3\varepsilon_0$ and $5\varepsilon_0$ ($\varepsilon_0>0$). One system is populated by spin $\frac{1}{2}$ fermions and the other by bosons. What is the value of E_f-E_B, where E_f and E_B are the ground state energies of the fermionic and bosonic systems respectively?

(a) $6\varepsilon_0$ (b) $2\varepsilon_0$ (c) $4\varepsilon_0$ (d) ε_0

55. The input to a lock in amplifier has the form $V_i(t) = V_i \sin(\omega t + \theta_i)$ where V_{i,ω,θ_i} are the amplitude, frequency and phase of the input signal respectively. This signal is multiplied by a reference signal of the same frequency ω, amplitude V_r and phase θ_r. If the multiplied signal is fed to a low pass filter of cut-off frequency ω, then the final output signal is

(a) $\frac{1}{2}V_iV_r\cos(\theta_i - \theta_r)$

(b) $V_iV_r\left[\cos(\theta_i - \theta_r) - \cos\left(\frac{1}{2}\omega t + \theta_i + \theta_r\right)\right]$

(c) $V_iV_r\sin(\theta_i - \theta_r)$

(d) $V_iV_r\left[\cos(\theta_i - \theta_r) + \cos\left(\frac{1}{2}\omega t + \theta_i + \theta_r\right)\right]$

56. The solution of the partial differential equation $\frac{\partial^2}{\partial t^2}u(x,t) - \frac{\partial^2}{\partial x^2}u(x,t) = 0$ satisfying the boundary conditions $u(0,t) = 0 = u(L,t)$ and initial conditions $u(x,0) = \sin(\pi x/L)$ and $\frac{\partial}{\partial x}u(x,t)|_{t=0} = \sin(2\pi x/L)$ is

(a) $\sin(\pi x/L)\cos(\pi t/L) + \frac{1}{2\pi}\sin(2\pi x/L)\cos(2\pi t/L)$

(b) $2\sin(\pi x/L)\cos(\pi t/L) - \sin(\pi x/L)\cos(2\pi t/L)$

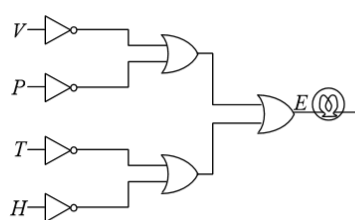
(c) $\sin(\pi x/L)\cos(2\pi t/L) + \frac{L}{\pi}\sin(2\pi x/L)\sin(\pi t/L)$

(d) $\sin(\pi x/L)\cos(\pi t/L) + \frac{1}{2\pi}\sin(2\pi x/L)\sin(2\pi t/L)$

57. Consider the hydrogen deuterium molecule HD. If the mean distance between the two atoms is 0.08 *nm* and the mass of the hydrogen atom is 938 MeV/c^2, then the energy difference ΔE between the two lowest rotational states is approximately

(a) $10^{-1}eV$ (b) $10^{-2}eV$ (3) $2\times10^{-2}eV$ (4) $10^{-3}eV$

58. Four digital outputs *V*, *P*, *T* and *H* monitor the speed v, tyre pressure *p*, temperature *t* and relative humidity *h* of a car. These outputs switch from 0 to 1 when the values of the parameters exceed 85 km/hr, 2 bar, 40^{o}C and 50%, respectively. A logic circuit that is used to switch ON a lamp at the output E is shown below.



which of the following conditions will switch the lamp ON?

(a) v < 85 km/hr, p < 2 bar, t > 40^{o}C, h > 50%

(b) v < 85 km/hr, p < 2 bar, t > 40^{o}C, h < 50%

(c) v > 85 km/hr, p < 2 bar, t > 40^{o}C, h < 50%

(d) v > 85 km/hr, p < 2 bar, t < 40^{o}C, h > 50%

59. The solutions of the differential equation $\frac{dx}{dt} = x^2$ with the initial condition $x(0) = 1$ will blow up as t tens to

(a) 1 (b) 2 (c) ½ (d) ∞

60. Let u be a random variable uniformly distributed in the interval [0,1] and $V = -c \ln u$, where c is a real constant. If V is to be exponentially distributed in the interval [0, ∞] with standard deviation, then the value of *c* should be

(a) ln2 (b) ½ (c) 1 (d) -1

61. The inverse Laplace transform of $\frac{1}{s^2(s+1)}$

(a) $\frac{1}{2}t^2e^{-t}$ (b) $\frac{1}{2}t^2+1-e^{-t}$ (c) $t-1+e^{-t}$ (d) $\frac{1}{2}t^2(1-e^{-t})$

62. The number of degrees of freedom of a rigid body in *d* space-dimensions is

(a) $2d$ (b) 6 (c) $\frac{d(d+1)}{2}$ (d) dt

63. A particle of mass m is at the stable equilibrium position of its potential energy, $V(x) = ax - bx^3$; where a, b are positive constants. The minimum velocity that has to be imparted to the particle to render its motion unstable is

(a) $(64a^2/9m^2b)^{1/4}$ (b) $(64a^3/27m^2b)^{1/4}$

(c) $(16a^3/27m^2b)^{1/4}$ (d) $(3a^3/64m^2b)^{1/4}$

64. If the operators A and B satisfy the commutation relation $[A,B] = I$, where I is the identity operator, then

(a) $[e^A, B] = e^A$ (b) $[e^A, B] = [e^B, A]$

(c) $[e^A, B] = [e^{-B}, A]$ (d) $[e^A, B] = I$

65. A system is governed by the Hamiltonian, $H = \frac{1}{2}(p_x - ay)^2 + \frac{1}{2}(p_y - bx)^2$; where a and b are constants and p_x, p_y are momenta conjugate to x and y respectively. For what values of a and b will the quantities $(p_x - 3y)$ and $(p_y + 2x)$ be conserved?

(a) $a = -3, b = 2$ (b) $a = 3, b = -2$

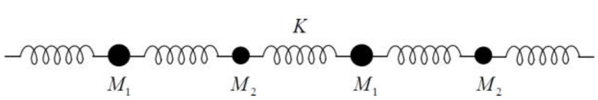
(c) $a = 2, b = -3$ (d) $a = -2, b = 3$

66. Using the frequency dependent Drude formula, what is the effective kinetic inductance of a metallic wire that is to be used as a transmission line? (in the following, the electrons mass is m, density of electrons is n, and the length and cross-sectional area of the wire and *l* and *A* respectively)

(a) $mA/(ne^2l)$ (b) zero

(c) $ml/(ne^2A)$ (d) $m\sqrt{A}/(ne^2l^2)$

67. The phonon dispersion for the following one-dimensional diatomic lattice with masses M_1 and M_2 (as shown in the figure)



is given by

$$\omega^2(q) = K\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\left[1 \pm \sqrt{1 - \frac{4M_1M_2}{(M_1+M_2)^2}\sin^2\left(\frac{qa}{2}\right)}\right]$$

where, 'a' is the lattice parameter and K is the spring constant. The velocity of sound is

(a) $\sqrt{\frac{K(M_1+M_2)}{2M_1M_2}}a$ (b) $\sqrt{\frac{K}{2(M_1+M_2)}}a$

(c) $\sqrt{\frac{K(M_1+M_2)}{M_1M_2}}a$ (d) $\sqrt{\frac{KM_1M_1}{2(M_1+M_2)}}a$

68. The binding energy of a light nucleus (Z, A) in MeV is given by the approximate formula, $B(A,Z)=16A-20A^{2/3}-\frac{3}{4}Z^2A^{-1/3}+30\frac{(N-Z)^2}{A}$; where $N=A-Z$ is the neutron number. The value of Z of the most stable isobar for a given A is

(a) $\frac{A}{2}\left(1-\frac{A^{2/3}}{160}\right)^{-1}$ (b) $\frac{A}{2}$

(c) $\frac{A}{2}\left(1-\frac{A^{2/3}}{120}\right)^{-1}$ (d) $\frac{A}{2}\left(1+\frac{A^{4/3}}{64}\right)$

69. Muons are produced through the annihilation of particle a and its antiparticle, namely the process, $a+\bar{a}\rightarrow\mu^{+}+\mu^{-}$. A muon has a rest mass of $105\,MeV/c^2$ and its proper life time is $2\mu S$. If the center of mass energy of the collision is 2.1 GeV in the laboratory frame that coincides with the centre of mass frame, then the fraction of muons that will decay before they reach a detector placed 6 km away from the interaction point is

(a) e^{-1} (b) $1-e^{-1}$ (c) $1-e^{-2}$ (d) e^{-10}

70. The conductors in a 0.75 km long two wire transmission line are separated by a centre to centre distance of 0.2 m. If each conductor has a diameter of 4 cm, then the capacitance of the line is

(a) $8.85\mu F$ (b) $88.5nF$ (c) $8.85pF$ (d) $8.85nF$

71. The electron dispersion relation for a one dimensional metal is given by

$$\varepsilon_k=2\varepsilon_0\left[\sin^2\frac{ka}{2}-\frac{1}{6}\sin^2 ka\right]$$

where, '$\boldsymbol{k}$' is the momentum, 'a' is the lattice constant, ε_0 is a constant having dimensions of energy and $[ka]\le\pi$. If the average number of electrons per atom in the conduction band is $1/3$, then the Fermi energy is

(a) $\varepsilon_0/4$ (b) ε_0 (c) $2\varepsilon_0/3$ (d) $5\varepsilon_0/3$

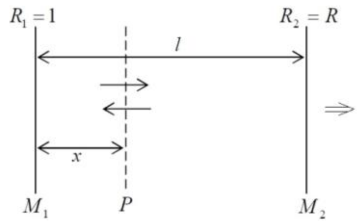
72. The electronic energy levels in a hydrogen atom are given by $\varepsilon_0=-13.6/n^2eV$. If a selective excitation to the $n=100$ level is to be made using a laser, the maximum allowed frequency line-width of the laser is

(a) 6.5 MHz (b) 6.5 GHz (c) 6.5 Hz (d) 6.5 kHz

73. If the energy dispersion of a two dimensional electron system is $E=u\hbar k$, where u is the velocity and k is the momentum, then the density of states $D(E)$ depends on the energy as

(a) $\frac{1}{\sqrt{E}}$ (b) $\sqrt{E}$ (c) E (d) constant

74. Consider the laser resonantor cavity shown in the figure. If I_1 is the intensity of the radiation at mirror M_1 and α is the gain coefficient of the medium between the mirrors, then the energy density of photons in the plane P at 'a' distance x from M_1 is



(a) $(I_1/c)e^{-\alpha x}$
(b) $(I_1/c)e^{\alpha x}$
(c) $(I_1/c)(e^{\alpha x} + e^{\alpha x})$
(d) $(I_1/c)e^{2\alpha x}$

75. A spin $\frac{1}{2}$ particle A undergoes the decay, $A \to B + C + D$; where R is known that B and C are also spin $\frac{1}{2}$ particles. The complete set of allowed values of the spin of the particle D is

(a) $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3....$
(b) 0, 1
(c) $\frac{1}{2}$ only
(d) $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, ...$