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LEADING INSTITUE FOR CSIR-JRF/NET, GATE & JAM CSIR-UGC-NET/JRF JUNE-2014

PHYSICAL SCIENCES JUNE 2014

PART-B

- 21. One gram of salt is dissolved in water that is filled to a height of 5cm in a beaker of diameter 10cm. The accuracy of length measurement is 0.01 cm while that of mass measurement is 0.01.1.1 mg. When measuring the concentration C, the fractional error $\Delta C/C$ is 1. 0.8% 2.0.14%
 - 3. 0.5% 4.0.28 %
- 22. A system can have three energy levels: $E = 0, \pm \varepsilon$. The level E=0 is doubly degenerate, while the others are non-degenerate. The average energy at inverse temperature β is
 - 2. $\frac{\varepsilon(e^{\beta\varepsilon} e^{-\beta\varepsilon})}{(1 + e^{\beta\varepsilon} + e^{-\beta\varepsilon})}$ 4. $-\varepsilon \tanh\left(\frac{\beta\varepsilon}{2}\right)$ 1. $-\varepsilon \tanh(\beta \varepsilon)$ 3. Zero
- 23. For a particular thermodynamic system the entropy S is related to the internal energy U and volume V by

 $S = cU^{3/4} V^{1/4}$ Where c is a constant. The Gibbs potential G = U - TS + pV for this system is $2. \frac{cU}{3}$ $4. \frac{US}{3}$

- 1. $\frac{3pU}{4T}$
- 3. Zero
- 24. An op amp-based voltage follower
 - 1. Is useful for converting a low impedance source into a high impedance source
 - 2. Is useful for converting a high impedance source into a low impedance source
 - 3. Has infinitely high closed loop output impedance
 - 4. Has infinitely high closed loop gain
- 25. A particle of mass m in three dimensions is in the potential

$$V(r) = \begin{cases} 0 & r < a \\ \infty & r \ge a \end{cases}$$

Its ground state energy is

1.
$$\frac{\pi^2 h^2}{2ma^2}$$

3. $\frac{3\pi^2 h^2}{2ma^2}$
2. $\frac{\pi^2 h^2}{ma^2}$
4. $\frac{9\pi^2 h^2}{2ma^2}$

1. Unaffected

26. Which of the graphs below gives the correct qualitative behavior of the energy density $E_T(\lambda)$ of blackbody radiation of wavelength α at two temperatures T_1 and T_2 ($T_1 < T_2$)?



With a, b > 0. The value of Ψ , when the system is in thermodynamic equilibrium is 1. Zero $2 \pm (a/6b)^{1/4}$ $3 \pm (a/3b)^{1/4}$ $4 \pm (a/b)^{1/4}$

30. The inner shield of a triaxial conductor is driven by an (ideal) op-amp follower circuit as shown. The effective capacitance between the signal-carrying conductor and ground is



31. Consider a system of two non-interacting identical fermions, each of mass m in an infinite square well potential of width a. (Take the potential inside the well to be zero and ignore spin.) the composite wave function for the system with total energy

$$E = \frac{5\pi^{2}h^{2}}{2ma^{2}} \text{ is}$$
1. $\frac{2}{a} \left[\sin\left(\frac{\pi x_{1}}{a}\right) \sin\left(\frac{2\pi x_{2}}{a}\right) - \sin\left(\frac{2\pi x_{1}}{a}\right) \sin\left(\frac{\pi x_{2}}{a}\right) \right]$
2. $\frac{2}{a} \left[\sin\left(\frac{\pi x_{1}}{a}\right) \sin\left(\frac{2\pi x_{2}}{a}\right) + \sin\left(\frac{2\pi x_{1}}{a}\right) \sin\left(\frac{\pi x_{2}}{a}\right) \right]$
3. $\frac{2}{a} \left[\sin\left(\frac{\pi x_{1}}{a}\right) \sin\left(\frac{3\pi x_{2}}{a}\right) - \sin\left(\frac{3\pi x_{1}}{a}\right) \sin\left(\frac{\pi x_{2}}{a}\right) \right]$
4. $\frac{2}{a} \left[\sin\left(\frac{\pi x_{1}}{a}\right) \sin\left(\frac{\pi x_{2}}{a}\right) - \sin\left(\frac{\pi x_{1}}{a}\right) \sin\left(\frac{\pi x_{2}}{a}\right) \right]$
32. A particle of mass m in the potential V(x,y) $= \frac{1}{2}m\omega^{2}(4x^{2} + y^{2})$, is in an eigen state of energy
$$E = \frac{5}{2}\hbar\omega. \text{ The corresponding un-normalized eigen function is}$$
1. $y \exp\left[-\frac{\pi \omega}{2h} (2x^{2} + y^{2}) \right]$
2. $x \exp\left[-\frac{m\omega}{2h} (2x^{2} + y^{2}) \right]$
3. $y \exp\left[-\frac{\pi \omega}{2h} (x^{2} + y^{2}) \right]$
4. $xy \exp\left[-\frac{\pi \omega}{2h} (x^{2} + y^{2}) \right]$

33. A particle of mass m and coordinate q has the Lagrangian

$$L = \frac{1}{2}m\dot{q^2} - \frac{\lambda}{2}q\dot{q^2}$$

Where λ is a constant? The Hamiltonian for the system is given by

1.
$$\frac{p^2}{2m} + \frac{\lambda q p^2}{2m^2}$$

3.
$$\frac{p^2}{2m} + \frac{\lambda q p^2}{2(m-\lambda q)^2}$$

4.
$$\frac{p\dot{q}}{2}$$

34. If $\vec{A} = yz\hat{\imath} + zx\hat{\jmath} + xy\hat{k}$ and C is the circle of unit radius in the plane defined by z=1, with the centre on the z-axis then the value of the integral $\oint_C \vec{A} \cdot \vec{dl}$ is

1.
$$\frac{\pi}{2}$$
 2. π 3. $\frac{\pi}{4}$ 4. 0

35. Given

$$\sum_{n=0}^{\infty} P_n(x) t^n = (1 - 2xt + t^2)^{-1/2}$$

For |t| < 1, the value of P₅(-1) is

1. 0.26 2.1 3.0.5 4.-1

36. A charged particle is at a distance d from an infinite conducting plane maintained at zero potential. When released from rest, the particle reaches a speed u at a distance d/2 from the plane. At what distance from the plane will the particle reach the speed 2u?

d/6
d/2
d/3
d/4

1.

37. Consider the matrix

$$M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$$

The Eigen values of M are

1. -5, -2, 72. -7, 0, 73. -4i, 2i, 2i4. 2, 3, 6

38. Consider the differential equation

- $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$ With the initial conditions x(0) = 01. 1/2 $\frac{x}{2.1}$ $\frac{x}{3.2}$ $\frac{x}{4.00} = 1$
- 39. A light source is switched on and off at a constant frequency f. An observer moving with a velocity u with respect to the light source will observe the frequency of the switching to be

1.
$$f(1-\frac{u^2}{c^2})^{-1}$$
 2. $f(1-\frac{u^2}{c^2})^{-1/2}$ 3. $f(1-\frac{u^2}{c^2})$ 4. $f(1-\frac{u^2}{c^2})^{1/2}$

40. If C is the contour defined by |z| = 1/2 the value of the integral

$$\oint_C \frac{dz}{\sin^2 z} is$$

$$\infty \qquad 2.2\pi i \qquad 3.0 \qquad 4.\pi i$$

- 41. The time period of a simple pendulum under the influence of the acceleration due to gravity g is T. the bob is subjected to an additional acceleration of magnitude $\sqrt{3}g$ in the horizontal direction. Assuming small oscillations, the mean position and time period of oscillation, respectively, of the bob will be
 - 1. 0° to the vertical and $\sqrt{3}T$ 2. 30° to the vertical and T/2
 - 3. 60° to the vertical and $T/\sqrt{2}$
- 4. 0° to the vertical and T/2
- 42. Consider an electromagnetic wave at the interface between two homogeneous dielectric media of dielectric constants ε_1 and ε_2 . Assuming $\varepsilon_2 > \varepsilon_1$ and no charges on the surface the electric field vector \vec{E} and the displacement vector \vec{D} in the two media satisfy the following inequalities.
 - 1. $|\overline{E_2}| > |\overline{E_1}|$ and $|\overline{D_2}| > |\overline{D_1}|$ 2. $|\overline{E_2}| < |\overline{E_1}|$ and $|\overline{D_2}| < |\overline{D_1}|$ 3. $|\overline{E_2}| < |\overline{E_1}|$ and $|\overline{D_2}| > |\overline{D_1}|$ 4. $|\overline{E_2}| > |\overline{E_1}|$ and $|\overline{D_2}| < |\overline{D_1}|$
- 43. If the electrostatic potential in spherical polar coordinates is

$$\varphi(\mathbf{r}) = \varphi_0 e^{-\mathbf{r}/\mathbf{r}}_{o}$$

Where φ_0 and r_0 are constants, then the charge density at a distance $r = r_0$ will be

1. $\frac{\varepsilon_0 \varphi_0}{er_0^2}$ 2. $\frac{e\varepsilon_0 \varphi_0}{2r_0^2}$ 3. $-\frac{\varepsilon_0 \varphi_0}{er_0^2}$ 4. $-\frac{2e\varepsilon_0 \varphi_0}{r_0^2}$

44. A current i_p flows through the primary coil of a transformer. The graph of $i_p(t)$ as a function of time t is shown in the figure below.



Which of the following graphs represents the current i_S in the secondary coil?



- 45. A time-dependent current $\vec{l}(t) = Kt\hat{z}$ (where K is a constant) is switched on at t =0 in an infinite current carrying wire. The magnetic vector potential at a perpendicular distance a from the wire is given (for time t > a/c) by
 - 1. $\hat{z} \frac{\mu_0}{4\pi c} \int_{-\sqrt{c^2 t^2 a^2}}^{\sqrt{c^2 t^2 a^2}} dz \frac{ct \sqrt{a^2 + z^2}}{(a^2 + z^2)^{1/2}}$ 2. $\hat{z} \frac{\mu_0 K}{4\pi} \int_{-ct}^{ct} dz \frac{t}{(a^2 + z^2)^{1/2}}$ 3. $\hat{z} \frac{\mu_0}{4\pi c} \int_{-ct}^{ct} dz \frac{ct - \sqrt{a^2 + z^2}}{(a^2 + z^2)^{1/2}}$ 4. $\hat{z} \frac{\mu_0}{4\pi c} \int_{-\sqrt{c^2 t^2 - a^2}}^{\sqrt{c^2 t^2 - a^2}} dz \frac{1}{(a^2 + z^2)^{1/2}}$
- 46. The pressure of a non-relativistic free Fermi gas in three dimensions depends, at T=0, on the density of fermions n as 1. $n^{5/3}$ 2. $n^{1/3}$ 3. $n^{2/3}$ 4. $n^{4/3}$
- 47. A double slit interference experiment uses a laser emitting light of two adjacent frequencies v_1 and $v_2(v_1 < v_2)$. The minimum path difference between the interfering beams for which the interference pattern disappears is
 - 1. $\frac{c}{v_2 + v_1}$ 2. $\frac{c}{v_2 v_1}$ 3. $\frac{c}{2(v_2 v_1)}$ 4. $\frac{c}{2(v_2 + v_1)}$

48. The recently-discovered Higgs boson at the LHC experiment has a decay mode into a photon and a Z boson. If the rest masses of the Higgs and Z boson are 125 GeV/c^2 and 90 GeV/c^2 respectively, and the decaying Higgs particle is at rest, the energy of the photon will approximately be

 1. $35\sqrt{3}$ GeV
 2. 35 GeV
 3. 30GeV
 4. 15GeV

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- 49. A permanently deformed even-even nucleus with $J^P = 2^+$ has rotational energy 93 keV. The energy of the next excited state is
 - 1. 372 keV 2. 310 keV

3. 273 keV

4. 186 keV

- 50. How much does the total angular momentum quantum number J change in the transition of $Cr(3d^6)$ atoms as it ionizes to $Cr^{2+}(3d^4)$?
 - 1. Increases by 2
 - 2. Decreases by 2
 - 3. Decreases by 4
 - 4. Does not change
- 51. For the logic circuit shown in the figure below



- 52. A spectral line due to a transition from an electronic state p to an s state splits into three Zeeman lines in the presence of a strong magnetic field. At intermediate field strengths the number of spectral lines is
 1. 10
 2. 3
 3. 6
 4. 9
- 53. A particle in the infinite square well

$$V(x) = \begin{cases} 0 & 0 < x < a \\ \infty & otherwise \end{cases}$$

Is prepared in a state with the wave function
$$\Psi(x) = \begin{cases} Asin^3 \left(\frac{\pi x}{a}\right) & 0 < x < a \\ 0 & otherwise \end{cases}$$

The expectation value of the energy of the particle is
$$1. \frac{5h^2 \pi^2}{2ma^2} \qquad 2. \frac{9h^2 \pi^2}{2ma^2} \qquad 3. \frac{9h^2 \pi^2}{10ma^2} \qquad 4. \frac{h^2 \pi^2}{2ma^2} \end{cases}$$

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- 54. The average local internal magnetic field acting on an Ising spin is $H_{int} = \alpha M$, where M is the magnetization and α is a positive constant. At a temperature T sufficiently close to (and above) the critical temperature T_c , the magnetic susceptibility at zero external field is proportional to (k_B is the Boltzmann constant) 2. $(k_{\rm B}T + \alpha)^{-1}$ 3. $(k_{\rm B}T - \alpha)^{-1}$ 4. Tanh(k_BT+ α) 1. $k_BT-\alpha$ 55. In one dimension, a random walker takes a step with equal probability to the left or right. What is the probability that the walker returns to the starting point after 4 steps? 4. 1/16 1. 3/8 2.5/16 3.1/4 56. Consider an electron in a b.c.c. lattice with lattice constant a. A single particle wave function that satisfies the Block theorem will have the form $f(\vec{r}) \exp(i\vec{k}.\vec{r})$, with $f(\vec{r})$ being $1 + \cos\left[\frac{2\pi}{a}(x + y - z)\right] + \cos\left[\frac{2\pi}{a}(-x + y + z)\right] + \cos\left[\frac{2\pi}{a}(x - y + z)\right]$ 1. $1 + \cos\left[\frac{2\pi}{a}(x+y)\right] + \cos\left[\frac{2\pi}{a}(y+z)\right] + \cos\left[\frac{2\pi}{a}(z+x)\right]$ 2. $1 + \cos\left[\frac{\pi}{a}(x+y)\right] + \cos\left[\frac{\pi}{a}(y+z)\right] + \cos\left[\frac{\pi}{a}(z+x)\right]$ 3. $1 + \cos\left[\frac{\pi}{a}(x+y-z) + \cos\left[\frac{\pi}{a}(-x+y+z)\right] + \cos\left[\frac{\pi}{a}(x-y+z)\right]\right]$ 4. 57. The dispersion relation for electrons in an f.c.c. crystal is given in the tight binding approximation by $\varepsilon(k) = -4\varepsilon_o \left| \cos\frac{k_x a}{2} \cos\frac{k_y a}{2} + \cos\frac{k_y a}{2} \cos\frac{k_2 a}{2} + \cos\frac{k_z a}{2} \cos\frac{k_z a}{2} \right|$ Where a is the lattice constant ε_0 is a constant with the dimension of energy. The x-component of the velocity of the electrons at $\left(\frac{\pi}{a}, 0, 0\right)$ is $1.-2\varepsilon_0 a/h$ $2\varepsilon_0 a/h$ $3.-4\varepsilon_0 a/h$ $4\varepsilon_0 a/h$ 58. The following data is obtained in an experiment that measures the viscosity η as a function of molecular weight M for a set of polymers M (Da) η (kPa-s) 990 0.28 ± 0.03 5032 30 ± 2 10191 250±10 19825 2000 + 200The relation that best describes the dependence of η on M is
 - 1. $\eta \sim M^{4/9}$ 2. $\eta \sim M^{3/2}$ 3. $\eta \sim M^2$ 4. $\eta \sim M^3$
- 59. The integral $\int_0^1 \sqrt{x} dx$ is to be evaluated up to 3 decimal places using Simpson's 3-point rule. If the interval [0,1] is divided into 4 equal parts, the correct result is 1. 0.683 2. 0.667 3. 0.657 4. 0.638



64. The function
$$\Phi(x, y, z, t) = \cos(z - vt) + Re(\sin(x + iy))$$
satisfies the equation
1. $\frac{1}{v^2} \frac{\partial^2 \Phi}{\partial t^2} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \Phi$
2. $\left(\frac{1}{v^2} \frac{\partial^2 \Phi}{\partial t^2} + \frac{\partial^2}{\partial z^2}\right) \Phi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \Phi$
3. $\left(\frac{1}{v^2} \frac{\partial^2 \Phi}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right) \Phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right) \Phi$
4. $\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v^2} \frac{\partial^2 \Phi}{\partial t^2}\right) \Phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right) \Phi$

- 65. The coordinates and momenta x_iP_i (i=1, 2, 3) of a particle satisfy the canonical Poisson bracket relations $\{x_i, p_j\} = \delta_{ij}$. If $C_1 = x_2p_3 + x_3p_2$ and $C_2 = x_1p_2 x_2p_1$ Are constants of motion, and if
 - $\begin{array}{l} C_3 = \{C_1, C_2\} = x_1p_3 + x_3p_1 \text{ then} \\ 1. \quad \{C_2, C_3\} = C_1 \text{ and } \{C_3, C_1\} = C_2 \\ 2. \quad \{C_2, C_3\} = -C_1 \text{ and } \{C_3, C_1\} = -C_2 \end{array}$
 - 3. $\{C_2, C_3\} = -C_1$ and $\{C_3, C_1\} = C_2$
 - 4. $\{C_2, C_3\} = C_1$ and $\{C_3, C_1\} = -C_2$
- 66. A canonical transformation relates the old coordinates (q, p) to the new ones (Q,P) by the relations $Q=q^2$ and P = p/2q. The corresponding time-independent generating function is 1. P/q^2 2. q^2P 3. q^2/P 4. qP^2
- 67. The time evolution of a one dimensional dynamical system is described by $\frac{dx}{dt} = -(x+1)(x^2 b^2)$

If this has one stable and two unstable fixed points, then the parameter b satisfies

- 1. 0 < b < 13. b < -14. b = 2
- 68. A charge (-e) is placed in vacuum at the point (d, 0, 0), where d > 0. The region $x \le 0$ is filled uniformly with a metal. The electric field at the point $(\frac{d}{2}, 0, 0)$ is

1.
$$-\frac{10e}{9\pi\varepsilon_0 d^2}$$
 (1,0,0)
3. $\frac{e}{\pi\varepsilon_0 d^2}$ (1,0,0)
4. $-\frac{e}{\pi\varepsilon_0 d^2}$ (1,0,0)

- 69. An electron is in the ground state of a hydrogen atom. The probability that it is with the Bohr radius is approximately equal to
 - 1. 0.60 2. 0.90 3. 0.16 4. 0.32
- 70. A beam of light of frequency ω is reflected from a dielectric –metal interface at normal incidence. The refractive index of the dielectric medium is n and that of the metal is $n_2 = n(1+ip)$. If the beam is polarized parallel to the interface, then the phase change experienced by the light upon reflection is 1. $\tan (2/p)$ 2. $\tan^{-1} (1/p)$ 3. $\tan^{-1}(2/p)$ 4. $\tan^{-1}(2p)$

71. The scattering amplitude $f(\theta)$ for the potential $V(r) = \beta e^{-\mu r}$, where β and μ are positive constants, is given in the Born approximation by

(In the following b= $2k\sin\frac{\theta}{2}$ and $E = \frac{\hbar^2 k^2}{2m}$) 1. $-\frac{4m\beta\mu}{\hbar^2(b^2+\mu^2)^2}$ 2. $-\frac{4m\beta\mu}{\hbar^2b^2(b^2+\mu^2)}$ 3. $-\frac{4m\beta\mu}{\hbar^2\sqrt{b^2+\mu^2}}$ 4. $-\frac{4m\beta\mu}{\hbar^2(b^2+\mu^2)^3}$

72. The ground state eigen function for the potential $V(x) = -\delta(x)$, where $\delta(x)$ is the delta function is given by $\Psi(x) = Ae^{-\alpha|x|}$, where A and $\alpha > 0$ are constants. If a perturbation H'=bx² is applied the first order correction to the energy of the ground state will be

1.
$$\frac{b}{\sqrt{2}\alpha^2}$$
 2. $\frac{b}{\alpha^2}$ 3. $\frac{2b}{\alpha^2}$ 4. $\frac{b}{2\alpha^2}$

73. A thin, infinitely long solenoid placed along the z-axis contains a magnetic flux Φ . Which of the following vector potentials corresponds to the magnetic field at an arbitrary point (x, y, z)?

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$$1. (A_x, A_y, A_z) = \left(-\frac{\Phi}{2\pi} \frac{y}{x^2 + y^2}, \frac{\Phi}{2\pi} \frac{x}{x^2 + y^2}, 0\right)$$

$$2. (A_x, A_y, A_z) = \left(-\frac{\Phi}{2\pi} \frac{y}{x^2 + y^2 + z^2}, \frac{\Phi}{2\pi} \frac{x}{x^2 + y^2 + z^2}, 0\right)$$

$$3. (A_x, A_y, A_z) = \left(-\frac{\Phi}{2\pi} \frac{x + y}{x^2 + y^2}, \frac{\Phi}{2\pi} \frac{x + y}{x^2 + y^2}, 0\right)$$

$$4. (A_x, A_y, A_z) = \left(-\frac{\Phi}{2\pi} \frac{y}{x^2 + y^2}, \frac{\Phi}{2\pi} \frac{y}{x^2 + y^2}, 0\right)$$

74. The van der Waals' equation of state for a gas is given by

$$\left(P+\frac{a}{v^2}\right)(V-b) = RT$$

Where P, V and T represent the pressure, volume and temperature respectively, and a and b are constants parameters. At the critical point, where all the roots of the above cubic equation are degenerate, the volume is given by

- 1. $\frac{a}{9b}$ 2. $\frac{a}{27b^2}$ 3. $\frac{8a}{27bR}$ 4. 3b
- 75. An electromagnetically –shielded room is designed so that at a frequency $\omega = 10^7 \text{ rad/s}$ the intensity of the external radiation that penetrates the room is 1% of the incident radiation. If $\sigma = \frac{1}{2\pi} X 10^6 (\Omega m)^{-1}$ is the conductivity of the shielding material, its minimum thickness should be (given that ln10=2.3) 1. 4.60 mm 2. 2.30 mm 3. 0.23 mm 4. 0.46 mm