

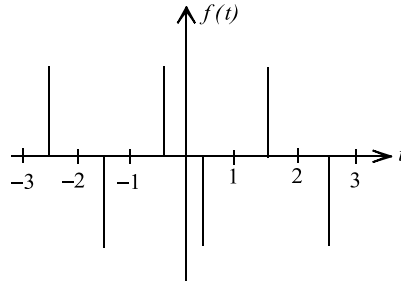
LEADING INSTITUTE FOR CSIR-JRF/NET, GATE & JAM

## CSIR-UGC-NET/JRF JUNE-2015

PHYSICAL SCIENCES JUNE 2015

### PART-B

21. Consider the periodic function  $f(t)$  with time period  $T$  as shown in the figure below.



The spikes, located at  $t = \frac{1}{2}(2n-1)$ , where  $n = 0, \pm 1, \pm 2, \dots$ , are Dirac-delta functions of strength  $\pm 1$ . The amplitudes  $a_n$  in the Fourier expansion

$$f(t) = \sum_{n=-\infty}^{\infty} a_n e^{2\pi i n t / T}$$

are given by

- (a)  $(-1)^n$  (b)  $\frac{1}{n\pi} \sin \frac{n\pi}{2}$  (c)  $i \sin \frac{n\pi}{2}$  (d)  $n\pi$

22. A particle moves in two dimensions on the ellipse  $x^2 + 4y^2 = 8$ . At a particular instant it is at the point  $(x, y) = (2, 1)$  and the  $x$ -components of its velocity are 6 (in suitable units). Then the  $y$ -components of its velocity is

- (a)  $-3$  (b)  $-2$  (c)  $1$  (d)  $4$

23. Consider the differential equation  $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0$ . If  $x = 0$  at  $t = 0$  and  $x = 1$  at  $t = 1$ , the value of  $x$  at  $t = 2$  is

- (a)  $e^2 + 1$  (b)  $e^2 + e$  (c)  $e + 2$  (d)  $2e$

24. The value of the integral  $\int_{-\infty}^{\infty} \frac{dx}{1+x^4}$  is

- (a)  $\frac{\pi}{\sqrt{2}}$  (b)  $\frac{\pi}{2}$  (c)  $\sqrt{2}\pi$  (d)  $2\pi$

25. The Laplace transform of  $6t^3 + 3\sin 4t$  is

- (a)  $\frac{36}{s^4} + \frac{12}{s^2 + 16}$  (b)  $\frac{36}{s^4} + \frac{12}{s^2 - 16}$  (c)  $\frac{18}{s^4} + \frac{12}{s^2 - 16}$  (d)  $\frac{36}{s^3} + \frac{12}{s^2 + 16}$

26. If the Lagrangian of a dynamical system in two dimensions is  $L = \frac{1}{2}m\dot{x}^2 + m\dot{x}\dot{y}$ , then its Hamiltonian is
- (a)  $H = \frac{1}{m}p_x p_y + \frac{1}{2m}p_y^2$  (b)  $H = \frac{1}{m}p_x p_y + \frac{1}{2m}p_x^2$   
 (c)  $H = \frac{1}{m}p_x p_y - \frac{1}{2m}p_y^2$  (d)  $H = \frac{1}{m}p_x p_y - \frac{1}{2m}p_x^2$
27. A particle of mass  $m$  moves in the one-dimensional potential  $V(x) = \frac{\alpha}{3}x^3 + \frac{\beta}{4}x^4$  where  $\alpha, \beta > 0$ . One of the equilibrium points is  $x=0$ . The angular frequency of small oscillations about the other equilibrium point is
- (a)  $\frac{2\alpha}{\sqrt{3m\beta}}$  (b)  $\frac{\alpha}{\sqrt{m\beta}}$  (c)  $\frac{\alpha}{\sqrt{12m\beta}}$  (d)  $\frac{\alpha}{\sqrt{24m\beta}}$
28. A particle of unit mass moves in the  $xy$ -plane in such a way that  $\dot{x}(t) = y(t)$  and  $\dot{y}(t) = -x(t)$ . We can conclude that it is in a conservative force-field which can be derived from the potential
- (a)  $\frac{1}{2}(x^2 + y^2)$  (b)  $\frac{1}{2}(x^2 - y^2)$  (c)  $x + y$  (d)  $x - y$
29. Consider three inertial frames of reference A, B and C. the frame B moves with a velocity  $c/2$  with respect to A and C moves with a velocity  $c/10$  with respect to B in the same direction. The velocity of C as measured in A is
- (a)  $\frac{3c}{7}$  (b)  $\frac{4c}{7}$  (c)  $\frac{c}{7}$  (d)  $\frac{\sqrt{3}c}{7}$
30. A plane electromagnetic wave is travelling along the positive  $z$ -direction. The maximum electric field along the  $x$ -direction is  $10 \text{ V/m}$ . The approximate maximum values of the power per unit area and the magnetic induction  $B$ , respectively, are
- (a)  $3.3 \times 10^{-7} \text{ watts / m}^2$  and  $10 \text{ tesla}$  (b)  $3.3 \times 10^{-7} \text{ watts / m}^2$  and  $3.3 \times 10^{-8} \text{ tesla}$   
 (c)  $0.265 \text{ watts / m}^2$  and  $10 \text{ tesla}$  (d)  $0.265 \text{ watts / m}^2$  and  $3.3 \times 10^{-8} \text{ tesla}$
31. Suppose the  $yx$ -plane forms a chargeless boundary between two media of permittivities  $\epsilon_{\text{left}}$  and  $\epsilon_{\text{right}}$  where  $\epsilon_{\text{left}} : \epsilon_{\text{right}} = 1 : 2$ . If the uniform electric field on the left is  $\vec{E}_{\text{left}} = c(\hat{i} + \hat{j} + \hat{k})$  (where  $c$  is a constant), then the electric field on the right  $\vec{E}_{\text{right}}$  is:
- (a)  $c(2\vec{i} + \vec{j} + \vec{k})$  (b)  $c(\vec{i} + 2\vec{j} + 2\vec{k})$  (c)  $c\left(\frac{1}{2}\vec{i} + \vec{j} + \vec{k}\right)$  (d)  $c\left(\vec{i} + \frac{1}{2}\vec{j} + \frac{1}{2}\vec{k}\right)$
32. A proton moves with a speed of  $300 \text{ m/s}$  in a circular orbit in the  $xy$ -plane in a magnetic field  $1 \text{ tesla}$  along the positive  $z$ -direction. When an electric field of  $1 \text{ V/m}$  is applied along the positive  $y$ -direction, the centre of the circular orbit
- (a) Remains stationary  
 (b) moves at  $1 \text{ m/s}$  along the negative  $x$ -direction  
 (c) Moves at  $1 \text{ m/s}$  along the positive  $z$ -direction  
 (d) Move at  $1 \text{ m/s}$  along the positive  $x$ -direction

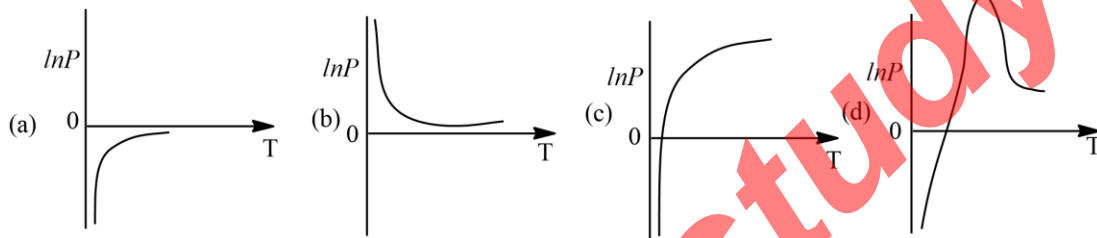
33. Which of the following transformations  $(V, \vec{A}) \rightarrow (V', \vec{A}')$  of the electrostatic potential  $V$  and the vector potential  $\vec{A}$  is a gauge transformation?
- (a)  $(V' = V + \alpha x, \vec{A}' = at \hat{k})$  (b)  $(V' = V + \alpha x, \vec{A}' = \vec{A} - at \hat{k})$   
 (c)  $(V' = V + \alpha x, \vec{A}' = \vec{A} + at \hat{i})$  (d)  $(V' = V + \alpha x, \vec{A}' = \vec{A} - at \hat{i})$
34. The ratio of the energy of the first excited state  $E_1$ , to that of the ground state  $E_0$  of a particle in a three-dimensional rectangular box of sides  $L, L$  and  $L/2$  is
- (a) 3:2 (b) 2:1 (c) 4:1 (d) 4:3
35. The waveform of a particle in one-dimension is denoted by  $\psi(x)$  in the coordinate representation and by  $\phi(p) = \int \psi(x) e^{-ipx/\hbar} dx$  in the momentum representation. If the action of an operator  $\hat{T}$  on  $\psi(x)$  is given by  $\hat{T}\psi(x) = \psi(x+a)$ , where  $a$  is a constant, then  $\hat{T}\phi(p)$  is given by
- (a)  $-\frac{i}{\hbar} ap \phi(p)$  (b)  $e^{-iap/\hbar} \phi(p)$  (c)  $e^{+iap/\hbar} \phi(p)$  (d)  $\left(1 + \frac{i}{\hbar} ap\right) \phi(p)$
36. If  $L_i$  are the components of the angular momentum operator  $\vec{L}$ , then the operator  $\sum_{i=1,2,3} [[\vec{L}, L_i], L_i]$  equals
- (a)  $\vec{L}$  (b)  $2\vec{L}$  (c)  $3\vec{L}$  (d)  $-\vec{L}$
37. A particle moves in one dimension in the potential  $V = \frac{1}{2}k(t)x^2$ , where  $k(t)$  is a time dependent parameter. Then  $\frac{d}{dt}\langle V \rangle$ , the rate of change of the expression value  $\langle V \rangle$  of the potential energy, is
- (a)  $\frac{1}{2} \frac{dk}{dt} \langle x^2 \rangle + \frac{k}{2m} \langle xp + px \rangle$  (b)  $\frac{1}{2} \frac{dk}{dt} \langle x^2 \rangle + \frac{1}{2m} \langle p^2 \rangle$   
 (c)  $\frac{k}{2m} \langle xp + px \rangle$  (d)  $\frac{1}{2} \frac{dk}{dt} \langle x^2 \rangle$
38. A system of  $N$  distinguishable particles, each of which can be in one of the two energy levels  $0$  and  $\varepsilon$ , has a total energy  $n\varepsilon$ , where  $n$  is an integer. The entropy of the system is proportional to
- (a)  $N \ln n$  (b)  $n \ln N$  (c)  $\ln \left( \frac{N!}{n!} \right)$  (d)  $\ln \left[ \frac{N!}{n!(N-n)!} \right]$
39. A system of  $N$  non-interacting classical particles, each of mass  $m$  is in a two-dimensional harmonic potential of the form  $V(r) = \alpha(x^2 + y^2)$  where  $\alpha$  is a positive constant. The canonical partition function of the system at temperature  $T$  is  $\left( \beta = \frac{1}{k_B T} \right)$
- (a)  $\left[ \left( \frac{\alpha}{2m} \right)^2 \frac{\pi}{\beta} \right]^N$  (b)  $\left( \frac{2m\pi}{\alpha\beta} \right)^{2N}$  (c)  $\left( \frac{\alpha\pi}{2m\beta} \right)^N$  (d)  $\left( \frac{2m\pi^2}{\alpha\beta^2} \right)^N$

40. In a two-state system, the transition rate of a particle from state 1 to state 2 is  $t_{12}$ , and the transition rate from state 2 to state 1 is  $t_{21}$ . In the steady state, the probability of finding the particle in state 1 is:

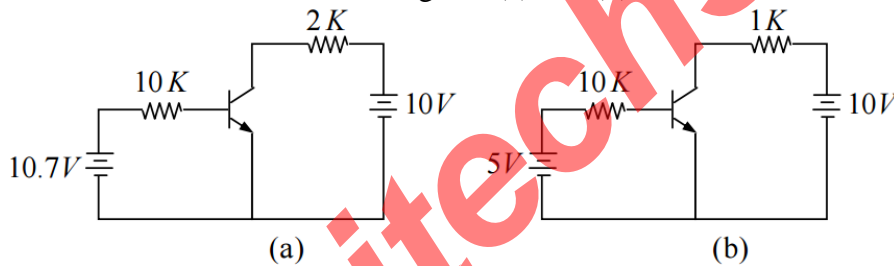
- (a)  $\frac{t_{21}}{t_{12} + t_{21}}$  (b)  $\frac{t_{12}}{t_{12} + t_{21}}$  (c)  $\frac{t_{12}t_{21}}{t_{12} + t_{21}}$  (d)  $\frac{t_{12} - t_{21}}{t_{12} + t_{21}}$

### PART-C

41. The condition for the liquid and vapour phases of a fluid to be in equilibrium is given by the approximate equation  $\frac{dP}{dT} \approx \frac{Q_1}{T v_{vap}}$  (Clausius-Clayperon equation), where  $v_{vap}$  is the volume per particle in the vapour phase, and  $Q_1$  is the latent heat, which may be taken to be a constant. If the vapour obeys ideal gas law, which of the following plots is correct?



42. Consider the circuits shown in figures (a) and (b) below.



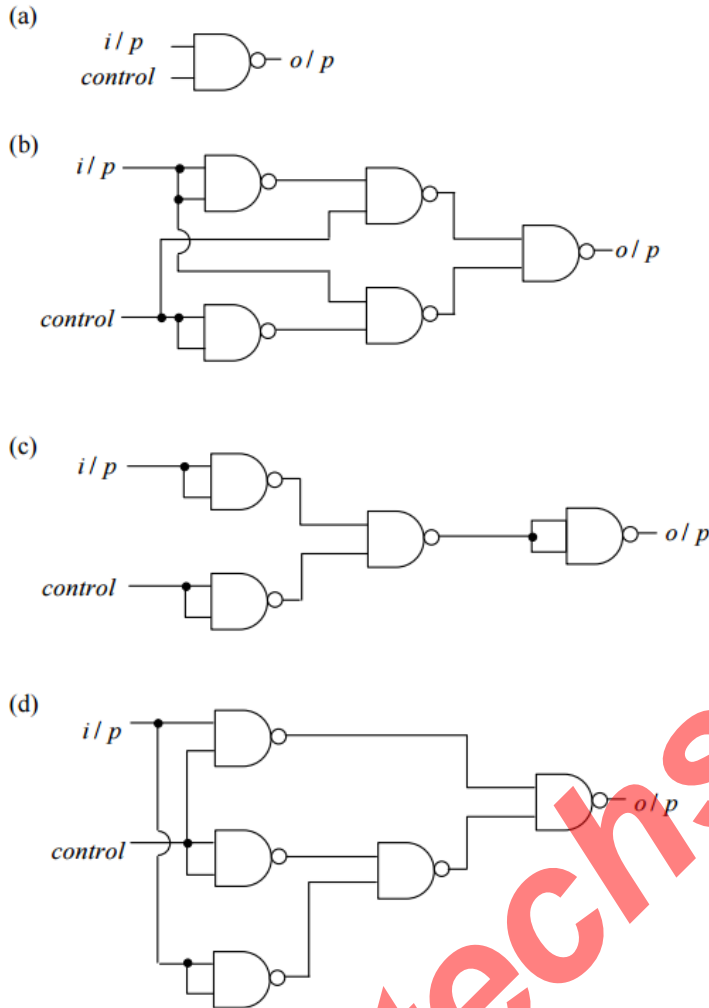
If the transistors in figures (a) and (b) have current gain ( $\beta_{dc}$ ) of 100 and 10 respectively, then they operate in the

- (a) Active region and saturation region respectively  
 (b) Saturation region and active region respectively  
 (c) Saturation region in both cases  
 (d) Active region in both cases

43. The viscosity  $\eta$  of a liquid is given by Poiscuille's formula  $\eta = \frac{\pi P a^4}{8 l V}$ . Assume that  $\ell$  and  $V$  can be measured very accurately, but the pressure  $P$  has an rms error of 1% and the radius  $a$  has an independent rms error of 3%. The rms error of the viscosity is closest to

- (a) 2% (b) 4% (c) 12% (d) 13%

44. Which of the following circuits behaves as a controlled inverter?



45. The concentration of electrons,  $n$  and holes,  $p$  for an intrinsic semiconductor at a temperature  $T$  can be expressed as  $n = p = AT^{3/2} \exp\left(-\frac{E_g}{2k_B T}\right)$ , where  $E_g$  is the band gap and  $A$  is a constant. If the mobility of both types of carriers is proportional to  $T^{-3/2}$ , then the log of the conductivity is a linear function of  $T^{-1}$  with slope

- (a)  $\frac{E_g}{(2k_B)}$  (b)  $\frac{E_g}{k_B}$  (c)  $\frac{-E_g}{(2k_B)}$  (d)  $\frac{-E_g}{k_B}$

46. Three real variables  $a$ ,  $b$  and  $c$  are each randomly chosen from a uniform probability distribution in the interval  $[0,1]$ . The probability that  $a+b > 2c$  is

- (a)  $\frac{3}{4}$  (b)  $\frac{2}{3}$  (c)  $\frac{1}{2}$  (d)  $\frac{1}{4}$

47. The rank-2 tensor  $x_i x_j$ , where  $x_i$  are the Cartesian coordinates of the position vector in three dimensions, has 6 independent elements. Under rotation, these 6 elements decompose into irreducible sets (that is the elements of each set transform only into linear combinations of elements in that set) containing

- (a) 4 and 2 elements (b) 5 and 1 elements  
(c) 3, 2 and 1 elements (d) 4, 1 and 1 elements

48. Consider the differential equation  $\frac{dy}{dx} = x^2 - y$  with the initial condition  $y = 2$  at  $x = 0$ . Let  $y_{(1)}$  and  $y_{(1/2)}$  be the solution at  $x = 1$  obtained using Euler's forward algorithm with step size 1 and  $\frac{1}{2}$  respectively. The value of  $(y_{(1)} - y_{(1/2)}) / y_{(1/2)}$  is

- (a)  $-\frac{1}{2}$  (b)  $-1$  (c)  $\frac{1}{2}$  (d)  $1$

49. Let  $f(x, t)$  be a solution of the wave equation  $\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$  in 1-dimension. If at  $t = 0$ ,  $f(x, 0) = e^{-x^2}$  and  $\frac{\partial f}{\partial t}(x, 0) = 0$  for all  $x$ , then  $f(x, t)$  for all future times  $t > 0$  is described by

- (a)  $e^{-(x^2 - v^2 t^2)}$  (b)  $e^{-(x - vt)^2}$  (c)  $\frac{1}{4} e^{-(x - vt)^2} + \frac{3}{4} e^{-(x + vt)^2}$  (d)  $\frac{1}{2} [e^{-(x - vt)^2} + e^{-(x + vt)^2}]$

50. Let  $q$  and  $p$  be the canonical coordinate and momentum of a dynamical system. Which of the following transformation is canonical?

$$A: Q_1 = \frac{1}{\sqrt{2}} q^2 \text{ and } P_1 = \frac{1}{\sqrt{2}} p^2$$

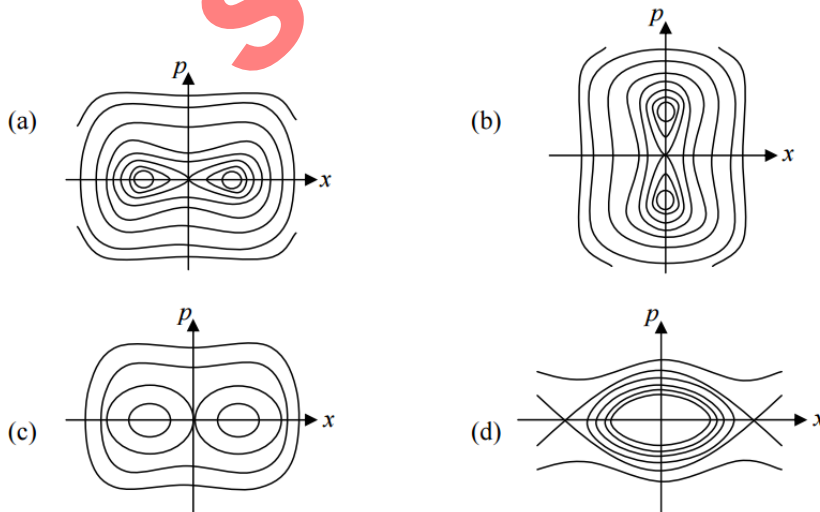
$$B: Q_2 = \frac{1}{\sqrt{2}} (p + q) \text{ and } P_2 = \frac{1}{\sqrt{2}} (p - q)$$

- (a) neither A nor B (b) both A and B (c) only A (d) only B

51. The differential cross section for scattering by a target is given by  $\frac{d\sigma}{d\Omega}(\theta, \varphi) = a^2 + b^2 \cos^2 \theta$ . If  $N$  is the flux of the incoming particles, the number of particles scattered per unit time is

- (a)  $\frac{4\pi}{3} N(a^2 + b^2)$  (b)  $4\pi N \left( a^2 + \frac{1}{6} b^2 \right)$   
 (c)  $4\pi M \left( \frac{1}{2} a^2 + \frac{1}{3} b^2 \right)$  (d)  $4\pi N \left( a^2 + \frac{1}{3} b^2 \right)$

52. Which of the following figures is a schematic representation of the phase space trajectories (i.e., contours of constant energy) of a particles moving in a one dimensional potential  $V(x) = -\frac{1}{2}x^2 + \frac{1}{4}x^4$ ?



53. Consider a rectangular wave guide with transverse dimensions  $2m \times 1m$  driven with an angular frequency  $\omega = 10^9 \text{ rad/s}$ . Which transverse electric (TE) modes will propagate in this wave guide?

- (a)  $TE_{10}, TE_{01}$  and  $TE_{20}$  (b)  $TE_{10}, TE_{11}$  and  $TE_{20}$   
 (c)  $TE_{01}, TE_{10}$  and  $TE_{11}$  (d)  $TE_{01}, TE_{10}$  and  $TE_{22}$

54. A rod of length  $L$  carries a total charge  $Q$  distributed uniformly. If this is observed in a frame moving with a speed  $v$  along the rod, the charge per unit length (as measured by the moving observer) is

- (a)  $\frac{Q}{L} \left(1 - \frac{v^2}{c^2}\right)$  (b)  $\frac{Q}{L} \sqrt{1 - \frac{v^2}{c^2}}$  (c)  $\frac{Q}{L \sqrt{1 - \frac{v^2}{c^2}}}$  (d)  $\frac{Q}{L \left(1 - \frac{v^2}{c^2}\right)}$

55. The electric and magnetic fields in the charge free region  $z > 0$  are given by

$$\vec{E}(\vec{r}, t) = E_0 e^{-k_1 z} \cos(k_2 x - \omega t) \hat{j}$$

$$B(r, t) = \frac{E_0}{\omega} e^{-k_1 z} \left[ k_1 \sin(k_2 x - \omega t) \hat{i} + k_2 \cos(k_2 x - \omega t) \hat{k} \right]$$

Where  $\omega, k_1$  &  $k_2$  are positive constants? The average energy flow in the  $x$ -direction is

- (a)  $\frac{E_0^2 k_2}{2\mu_0 \omega} e^{-2k_1 z}$  (b)  $\frac{E_0^2 k_2}{\mu_0 \omega} e^{-2k_1 z}$  (c)  $\frac{E_0^2 k_1}{2\mu_0 \omega} e^{-2k_1 z}$  (d)  $\frac{1}{2} c \epsilon_0 E_0^2 e^{-2k_1 z}$

56. A uniform magnetic field in the positive  $z$ -direction passes through a circular wire loop of radius 1 cm and resistance  $1\Omega$  lying in the  $xy$ -plane. The field strength is reduced from 10 tesla to 9 tesla in 1s. The charge transferred across any point in the wire is approximately

- (a)  $3.1 \times 10^{-4}$  coulomb (b)  $3.4 \times 10^{-4}$  coulomb  
 (c)  $4.2 \times 10^{-4}$  coulomb (d)  $5.2 \times 10^{-4}$  coulomb

57. The Dirac Hamiltonian  $H = c\vec{\alpha} \cdot \vec{p} + \beta mc^2$  for a free electron corresponds to the classical relation

$$E^2 = p^2 c^2 + m^2 c^4. \text{ The classical energy-momentum relation of a particle of charge } q \text{ in a}$$

electromagnetic potential  $(\phi, \vec{A})$  is  $(E - q\phi)^2 = c^2 \left( \vec{p} - \frac{q}{c} \vec{A} \right)^2 + m^2 c^4$ . Therefore the Dirac Hamiltonian for an electron in an electromagnetic field is

- (a)  $c\vec{\alpha} \cdot \vec{p} + \frac{e}{c} \vec{A} \cdot \vec{A} + \beta mc^2 - e\phi$  (b)  $c\vec{\alpha} \left( \vec{p} + \frac{e}{c} \vec{A} \right) + \beta mc^2 + e\phi$   
 (c)  $c \left( \vec{\alpha} \cdot \vec{p} + e\phi + \frac{e}{c} |\vec{A}| \right) + \beta mc^2$  (d)  $c\vec{\alpha} \cdot \left( \vec{p} + \frac{e}{c} \vec{A} \right) + \beta mc^2 - e\phi$

58. A particle of mass  $m$  is in a potential  $V = \frac{1}{2} m \omega^2 x^2$ , where  $\omega$  is a constant. Let  $\hat{a} = \sqrt{\frac{m\omega}{2\hbar}} \left( \hat{x} + \frac{i\hat{p}}{m\omega} \right)$ . In the

Heisenberg picture  $\frac{d\hat{a}}{dt}$  is given by

- (a)  $\omega \hat{a}$  (b)  $-i\omega \hat{a}$  (c)  $\omega \hat{a}^\dagger$  (d)  $i\omega \hat{a}^\dagger$

59. A particle of energy  $E$  scatters off a repulsive spherical potential

$$V(r) = \begin{cases} V_0 & \text{for } r < a \\ 0 & \text{for } r \geq a \end{cases}$$

Where  $V_0$  and  $a$  are positive constants. In the low energy limit, the total scattering, cross section is

$\sigma = 4\pi a^2 \left( \frac{1}{ka} \tanh ka - 1 \right)^2$ , where  $k^2 = \frac{2m}{\hbar^2}(V_0 - E) > 0$ . In the limit  $V_0 \rightarrow \infty$  the ratio of  $\sigma$  to the classical scattering cross-section off a sphere of radius  $a$  is

- (a) 4 (b) 3 (c) 1 (d)  $\frac{1}{2}$

60. Two different sets of orthogonal basis vectors  $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$  and  $\left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$  are given for a two-dimensional real vector space. The matrix representation of a linear operator  $\hat{A}$  in these bases are related by a unitary transformation. The unitary matrix may be chosen to be

- (a)  $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$  (b)  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  (c)  $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$  (d)  $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

61. A larger number  $N$  of Brownian particles in one-dimension start their diffusive motion from the origin at time  $t=0$ . The diffusion coefficient is  $D$ . The number of particles crossing a point at a distance  $L$  from the origin per unit time, depends on  $L$  and time  $t$  as

- (a)  $\frac{N}{\sqrt{4\pi Dt}} e^{-L^2/(4Dt)}$  (b)  $\frac{N}{\sqrt{4\pi Dt}} e^{-4Dt/L^2}$  (c)  $\frac{N}{\sqrt{16\pi Dt^3}} e^{-L^2/(4Dt)}$  (d)  $Ne^{-4Dt/L^2}$

62. Consider three Ising spins at the vertices of a triangle which interact with each other with a ferromagnetic Ising interaction of strength  $J$ . The partition function of the system at temperature  $T$  is given by  $\left( \beta = \frac{1}{k_B T} \right)$ :

- (a)  $2e^{3\beta J} + 6e^{-\beta J}$  (b)  $2e^{-3\beta J} + 6e^{\beta J}$   
 (c)  $2e^{3\beta J} + 6e^{-3\beta J} + 3e^{\beta J} + 3e^{-\beta J}$  (d)  $(2 \cosh \beta J)^3$

63. An ideal Bose gas in  $d$ -dimensions obeys the dispersion relation  $\varepsilon(\hat{k}) = Ak^s$ , where  $A$  and  $s$  are constants.

For Bose-Einstein condensation to occur, the occupancy of excited states

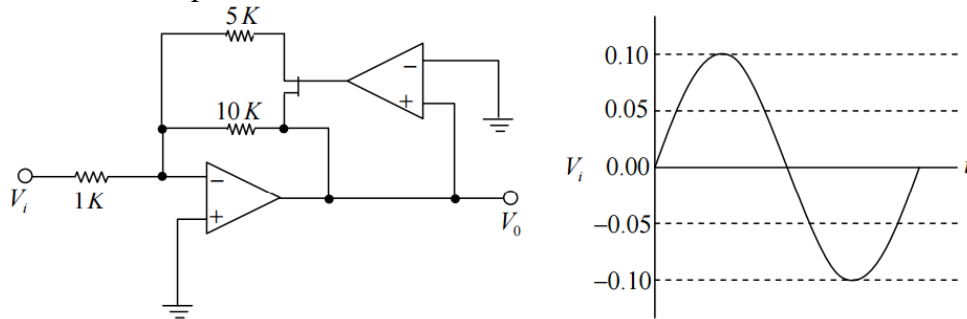
$$N_e = c \int_0^\infty \frac{\varepsilon^{(d-s)/s}}{e^{\beta(\varepsilon-\mu)} - 1} d\varepsilon$$

Where  $c$  is a constant, should remain finite even for  $\mu=0$ . This can happen if

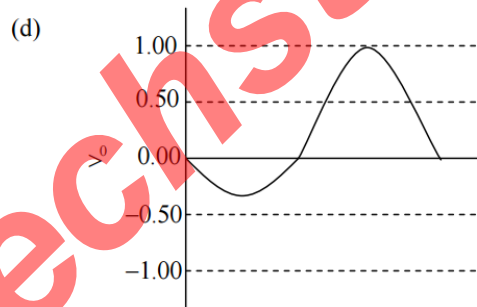
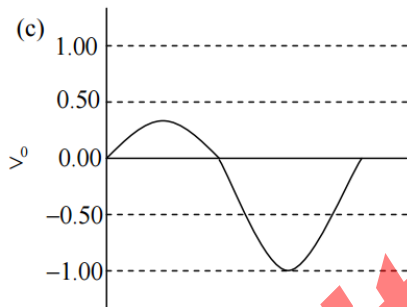
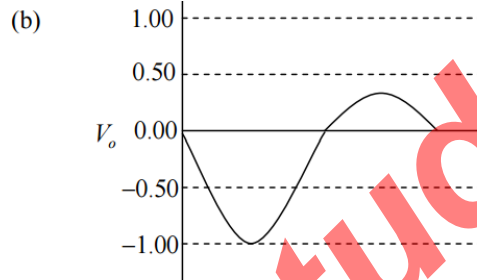
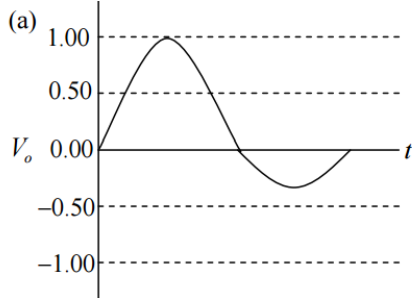
- (a)  $\frac{d}{s} < \frac{1}{4}$  (b)  $\frac{1}{4} < \frac{d}{s} < \frac{1}{2}$  (c)  $\frac{d}{s} > 1$  (d)  $\frac{1}{2} < \frac{d}{s} < 1$



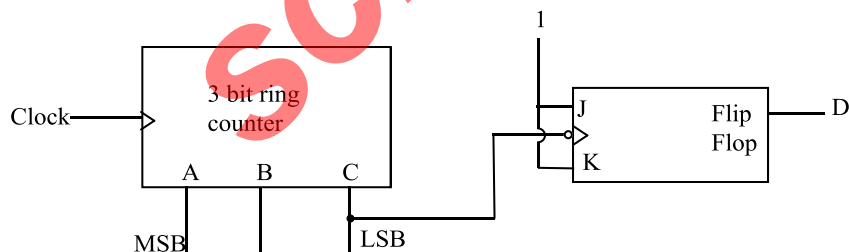
64. For the circuit and the input sinusoidal waveform shown in the figures below, which is the correct wave form at the output?



(The time scales in all the plots are the same).

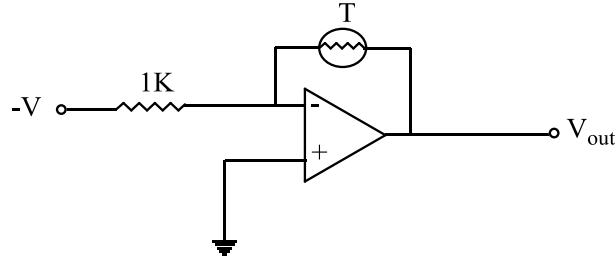


65. For the logic circuit given below, the decimal count sequence and the modulus of the circuit corresponding to A B C D are



- (a)  $8 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow 9 \rightarrow 5 \pmod{6}$  (b)  $8 \rightarrow 4 \rightarrow 2 \rightarrow 9 \rightarrow 5 \rightarrow 3 \pmod{6}$   
 (c)  $2 \rightarrow 5 \rightarrow 9 \rightarrow 1 \rightarrow 3 \pmod{5}$  (d)  $8 \rightarrow 5 \rightarrow 1 \rightarrow 3 \rightarrow 7 \pmod{5}$

66. In the circuit given below, the thermistor has a resistance  $3k\Omega$  at  $25^\circ C$ . Its resistance decreases by  $150\Omega$  per  $^\circ C$  upon heating. The output voltage of the circuit at  $30^\circ C$  is



- (a)  $-3.75V$  (b)  $-2.25V$  (c)  $2.25V$  (d)  $3.75V$
67. The low-energy electronic excitations in a two-dimensional sheet of graphene is given by  $E(\vec{k}) = \hbar v k$ , where  $v$  is the velocity of the excitations. The density of states is proportional to
- (a)  $E$  (b)  $E^{3/2}$  (c)  $E^{1/2}$  (d)  $E^2$
68. X-ray of wavelength  $\lambda = a$  is reflected from the (1 1 1) plane of a simple cubic lattice. If the lattice constant is  $a$ , the corresponding Bragg angle (in radian) is
- (a)  $\frac{\pi}{6}$  (b)  $\frac{\pi}{4}$  (c)  $\frac{\pi}{3}$  (d)  $\frac{\pi}{8}$
69. The critical magnetic field of a super conductor at temperatures 4 K and 8 K are 11 mA/m and 5.5 mA/m respectively. The transition temperature is approximately
- (a) 8.4 K (b) 10.6 K (c) 12.9 K (d) 15.0 K
70. A diatomic molecule has vibration states with energies  $E_v = \hbar\omega\left(v + \frac{1}{2}\right)$  and rotational states with energies  $E_j = B j(j+1)$ , where  $v$  and  $j$  are non-negative integers. Consider the transitions in which both the initial and final states are restricted to  $v \leq 1$  and  $j \leq 2$  and subject to the selection rules  $\Delta v = \pm 1$  and  $\Delta j = \pm 1$ . Then the largest allowed energy of transition is
- (a)  $\hbar\omega - 3B$  (b)  $\hbar\omega - B$  (c)  $\hbar\omega + 4B$  (d)  $2\hbar\omega + B$
71. Of the following term symbols of the  $np^2$  atomic configurations,  $^1S_0$ ,  $^3P_0$ ,  $^3P_1$ ,  $^3P_2$  and  $^1D_2$  which is the grounded state?
- (a)  $^3P_0$  (b)  $^1S_0$  (c)  $^3P_2$  (d)  $^3P_1$
72. A He-Ne laser operates by using two energy levels of Ne separated by 2.26 eV. Under steady state conditions of optical pumping, the equivalent temperature of the system at which the ratio of the number of atoms in the upper state to that in the lower state will be 1/20, is approximately (the Boltzmann constant  $k_B = 8.6 \times 10^{-5} \text{ eV/K}$ )
- (a)  $10^{10} K$  (b)  $10^8 K$  (c)  $10^6 K$  (d)  $10^4 K$
73. Let us approximate the nuclear potential in the shell model by 3-dimensional isotropic harmonic oscillator. Since the lowest two energy levels have angular momenta  $l=0$  and  $l=1$  respectively, which of the following two nuclei have magic numbers of protons and neutrons?
- (a)  $^4_2\text{He}$  and  $^{16}_8\text{O}$  (b)  $^2_1\text{D}$  and  $^8_4\text{Be}$  (c)  $^4_2\text{He}$  and  $^8_4\text{Be}$  (d)  $^4_2\text{He}$  and  $^{12}_6\text{Be}$

74. The charm quark is assigned a charm quantum number  $C=1$ . How should the Gellmann-Nishijima formula for electric charge be modified for four flavours of quarks?

- (a)  $I_3 + \frac{1}{2}(B - S - C)$                       (b)  $I_3 + \frac{1}{2}(B - S + C)$   
(c)  $I_3 + \frac{1}{2}(B + S - C)$                       (d)  $I_3 + \frac{1}{2}(B + S + C)$

75. The reaction  ${}^2_1D + {}^2_1D \rightarrow {}^4_2H + \pi^0$  cannot proceed via strong interactions because it violates the conservation of

- (a) angular momentum              (b) electric charge              (c) baryon number              (d) isospin

scitechstudy